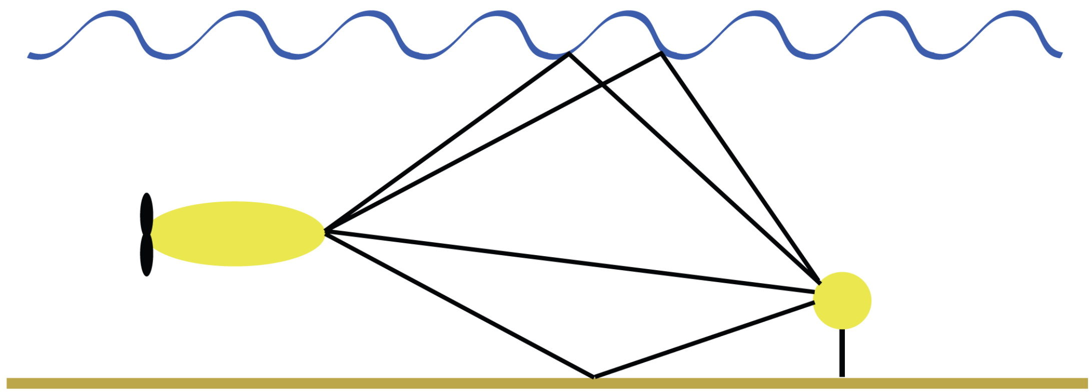
Communicating through the Ocean: Introduction and Challenges



Ballard Blair

bjblair@mit.edu

PhD Candidate

MIT/WHOI Joint Program

Advisor: Jim Presisig

Background

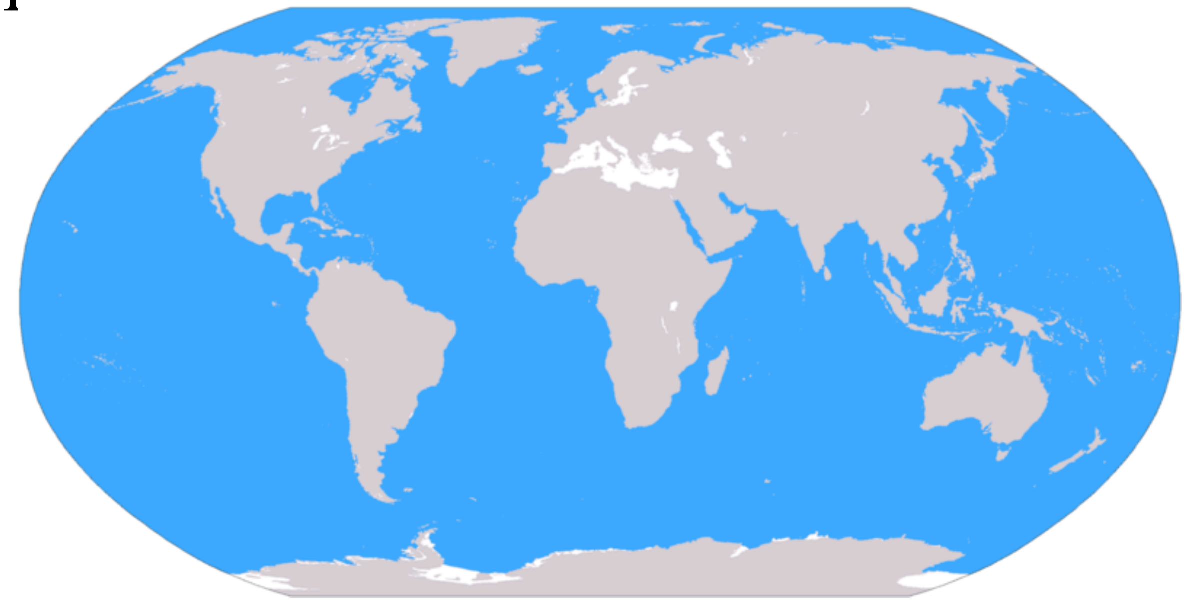
- BS in Electrical and Computer Engineering, Cornell university 2002
- MS in Electrical and Computer Engineering, Johns Hopkins 2005
- Hardware Engineer, JHUAPL 2002-2005
- PhD Candidate, MIT/WHOI Joint Program

Goals of Talk

- Motivate need for wireless underwater communication research
- Introduce difficulties of underwater acoustic communications
- Discuss current methods for handling underwater channel

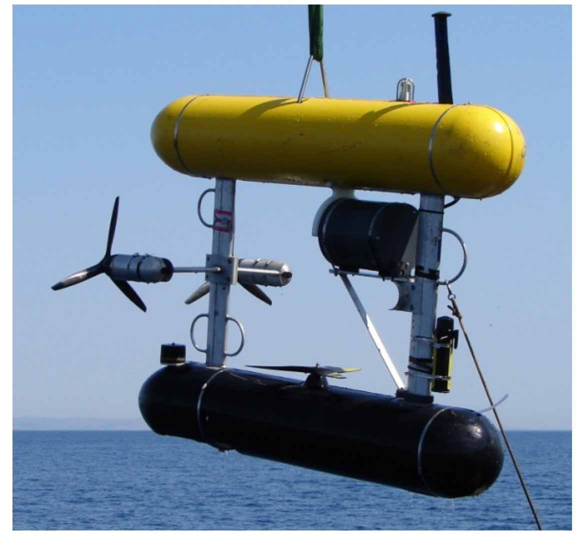
Introduction and Motivation

- Ocean covers over 70% of planet
- 11,000 meters at deepest point
- Ocean is 3-dimensional
- Only 2-3% explored



Communications in the ocean

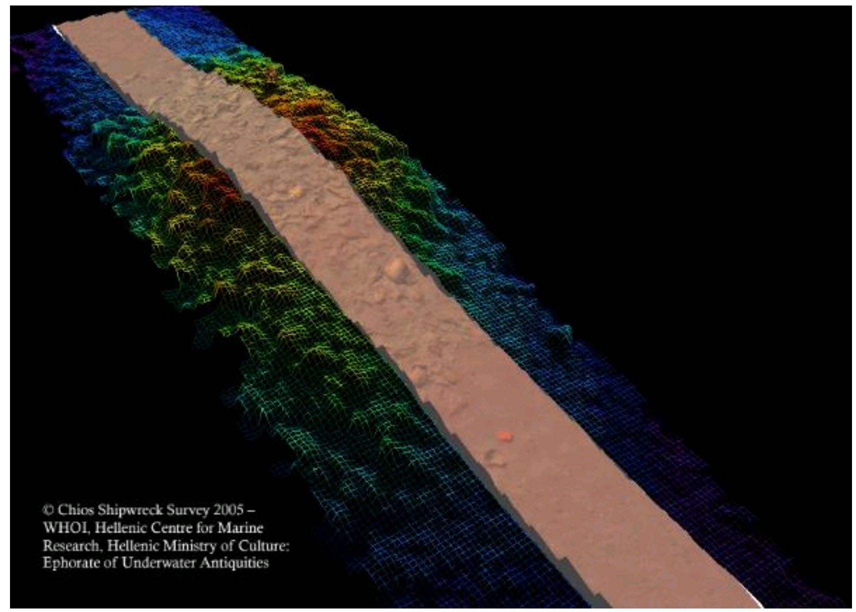
- Instruments / Sensor networks



- Gliders
- Manned Vehicles
- Unmanned underwater vehicles (UUV)
  - Autonomous underwater vehicles (AUV)
  - Remotely operated vehicles (ROV)
  - Hybrid underwater vehicles (H-AUV / H-ROV)

Current Applications

- Science
  - Geological / bathymetric surveys
  - Underwater archeology
  - Ocean current measurement
  - Deep ocean exploration
- Government
  - Fish population management
  - Costal inspection
  - Harbor safety
- Industry
  - Oil field discovery/maintenance



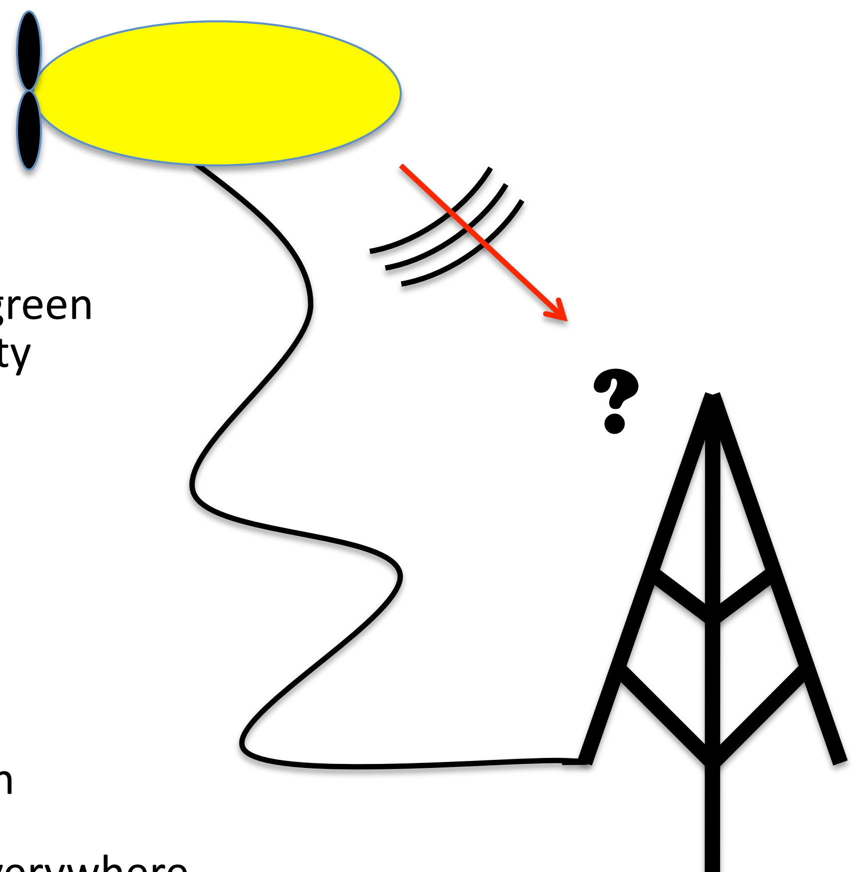
WHOI, 2005

Applications planned / in development

- Ocean observation system
  - Costal observation
- Military
  - Submarine communications (covert)
  - Ship inspection
- Networking
  - Mobile sensor networks (DARPA)
- Vehicle deployment
  - Multiple vehicles deployed simultaneously
  - Resource sharing among vehicles

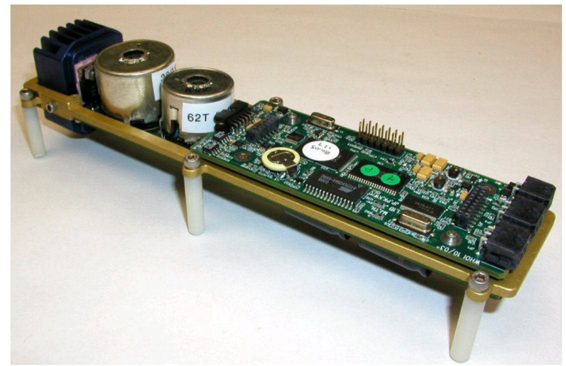
Technology for communication

- Radio Frequency (~1m range)
  - Absorbed by seawater
- Light (~100m range)
  - Hard to aim/control
  - High attenuation except for blue/green
  - Strong dependence on water clarity
- Ultra Low Frequency (~100 km)
  - Massive antennas (miles long)
  - Very narrowband (~50 Hz)
  - Not practical outside of navy
- Cable
  - Expensive/hard to deploy maintain
  - Impractical for mobile work sites
  - Ocean is too large to run cables everywhere
  - Can't run more than one cable from a ship

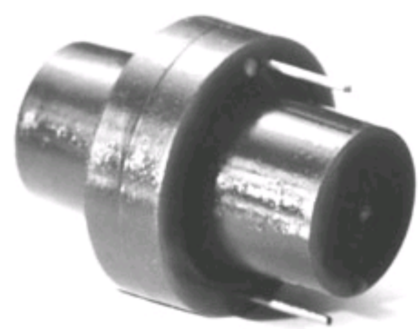


The Solution: Acoustics

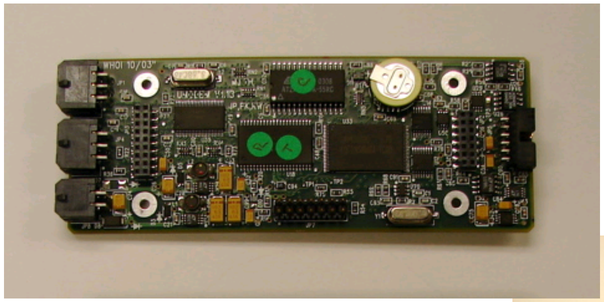
- Fairly low power
  - ~10-100W Tx
  - ~100 mW Rx
- Well studied
  - Cold war military funding
- Compact
  - Small amount of hardware needed
- Current Best Solution



WHOI Micromodem

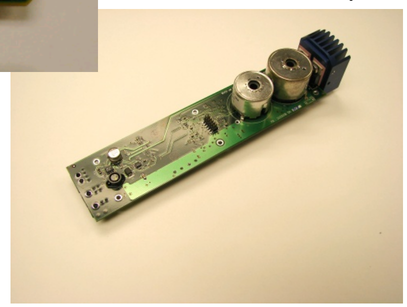


Example Hardware

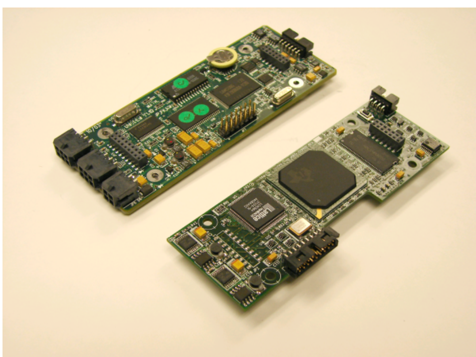


WHOI Micromodem

Power Amp



Micromodem in action

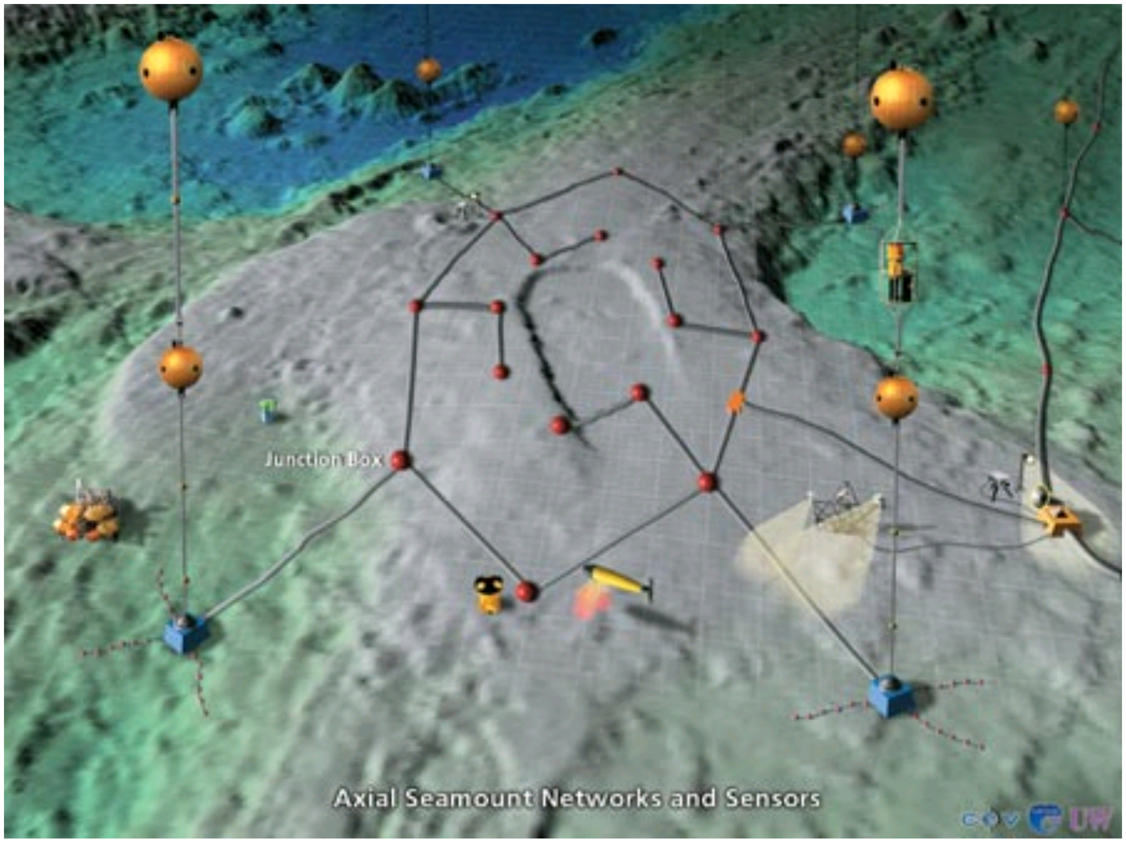


Daughter Card / Co-processor

Micromodem Specifications

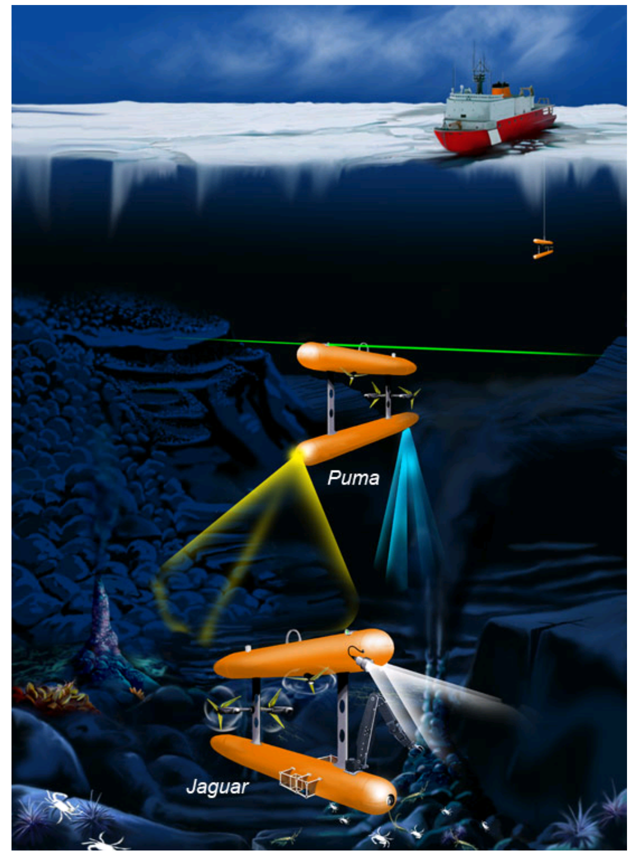
DSP	Texas Instruments TMS320C5416 100MHz low-power fixed point processor
Transmit Power	10 Watts Typical match to single omni-directional ceramic transducer.
Receive Power	80 milliwatts While detecting or decoding an low rate FSK packet.
Data Rate	80-5400 bps 5 packet types supported. Data rates higher than 80bps FSK require additional co-processor card to be received.

Underwater Technology

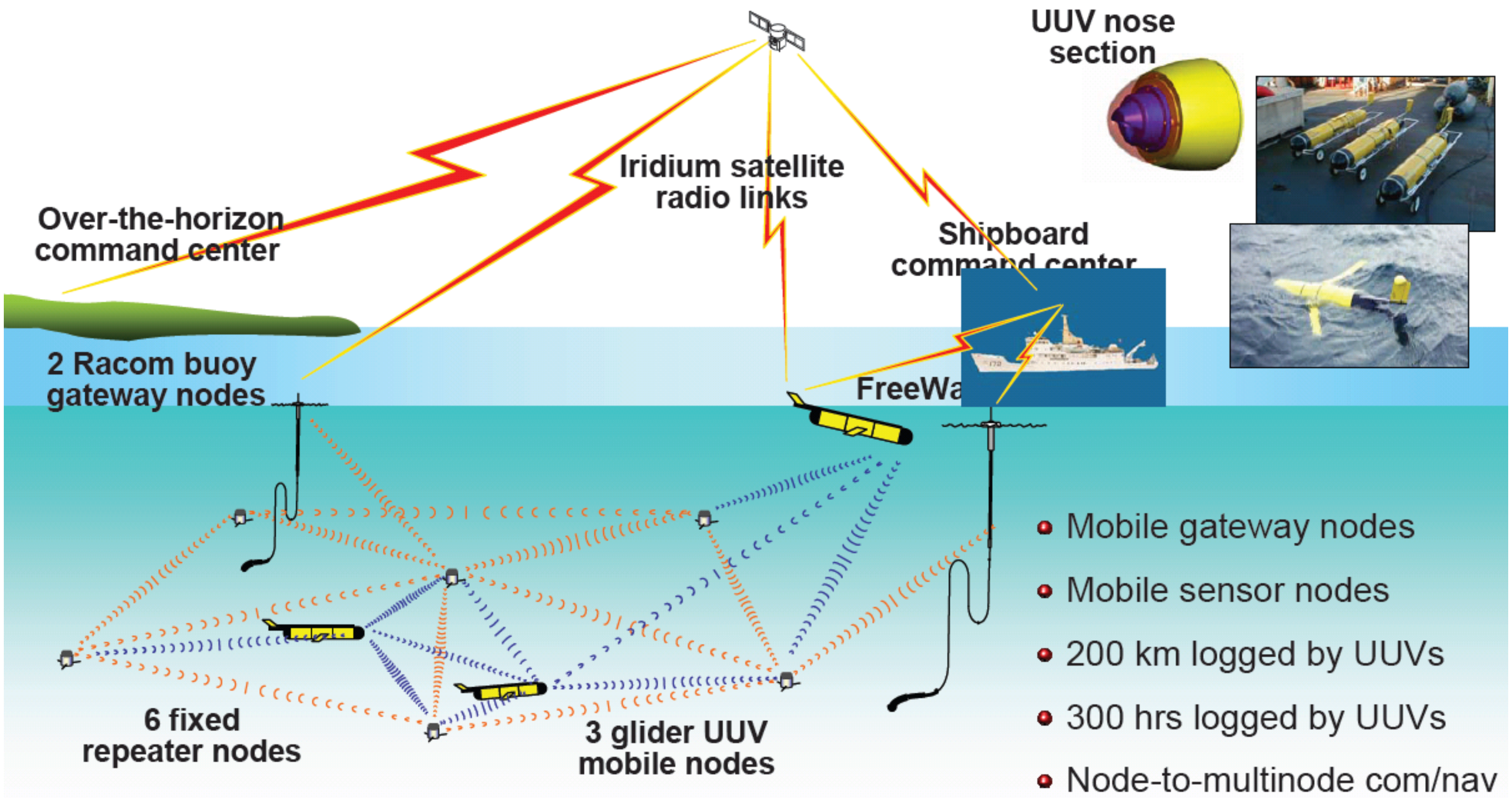


NEPTUNE Regional Observatory

WHOI, 2006



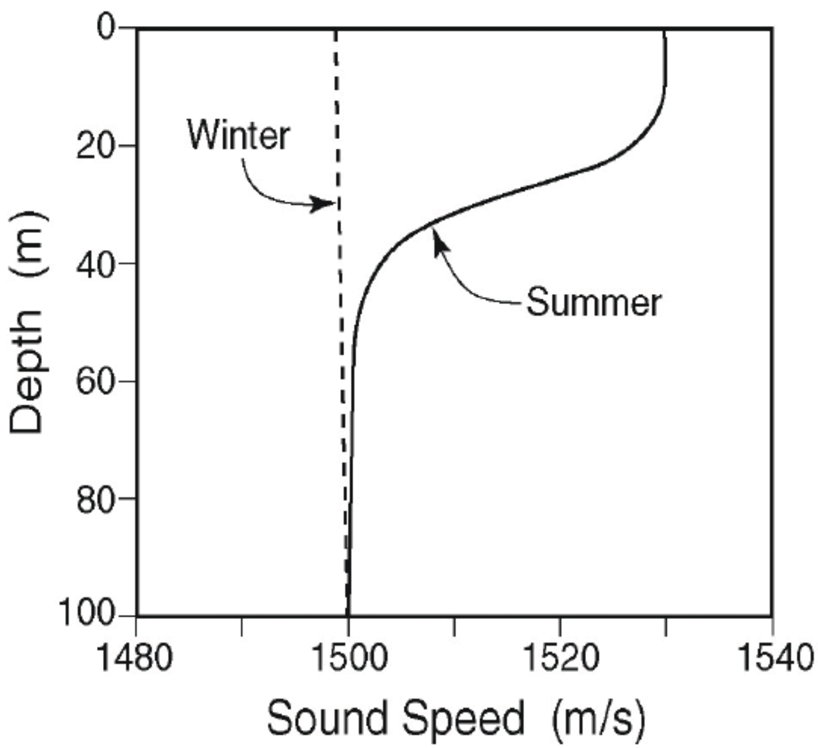
Example Communication System?

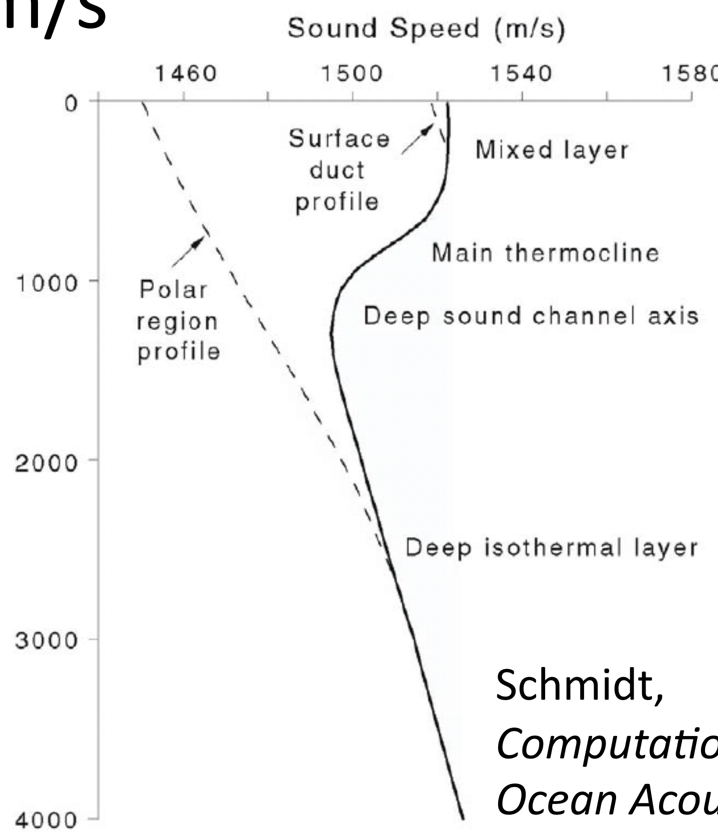


PLUSnet/Seaweb

Sound Profile

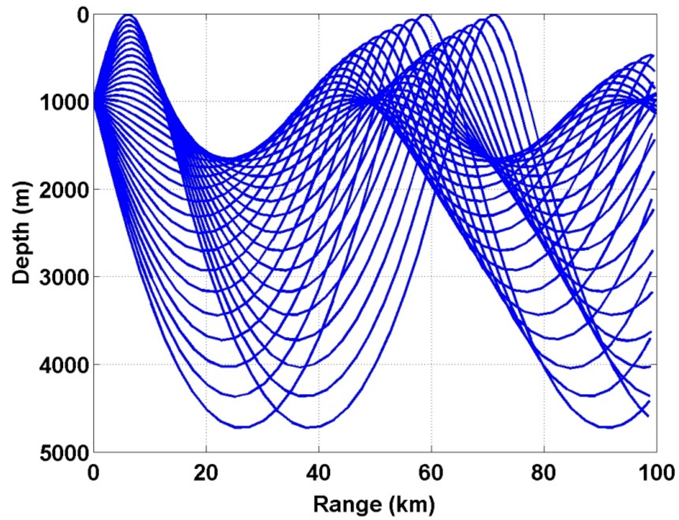
- Speed of sound ~ 1500 m/s
- Speed of light ~ 3x10^8 m/s

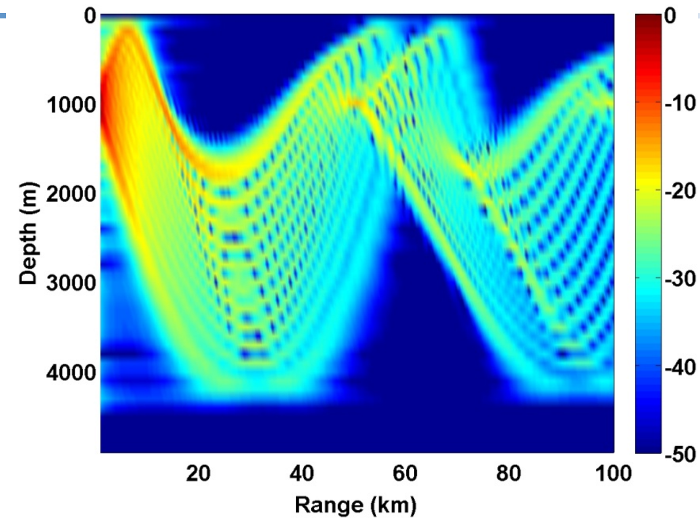


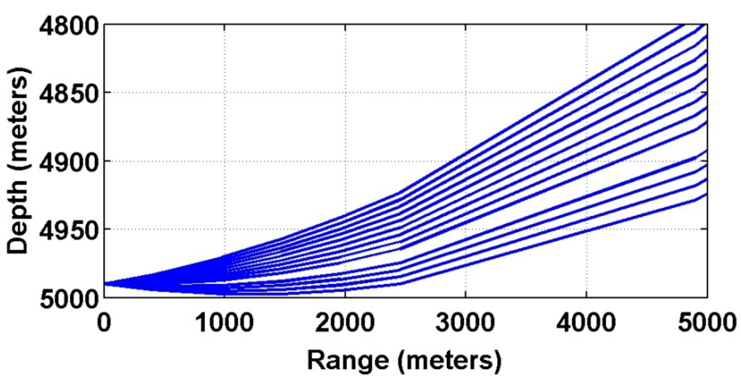


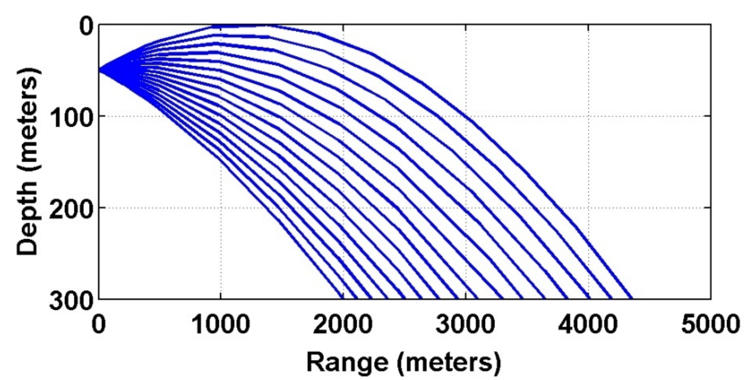
Schmidt, *Computational Ocean Acoustics*

Shadow Zones





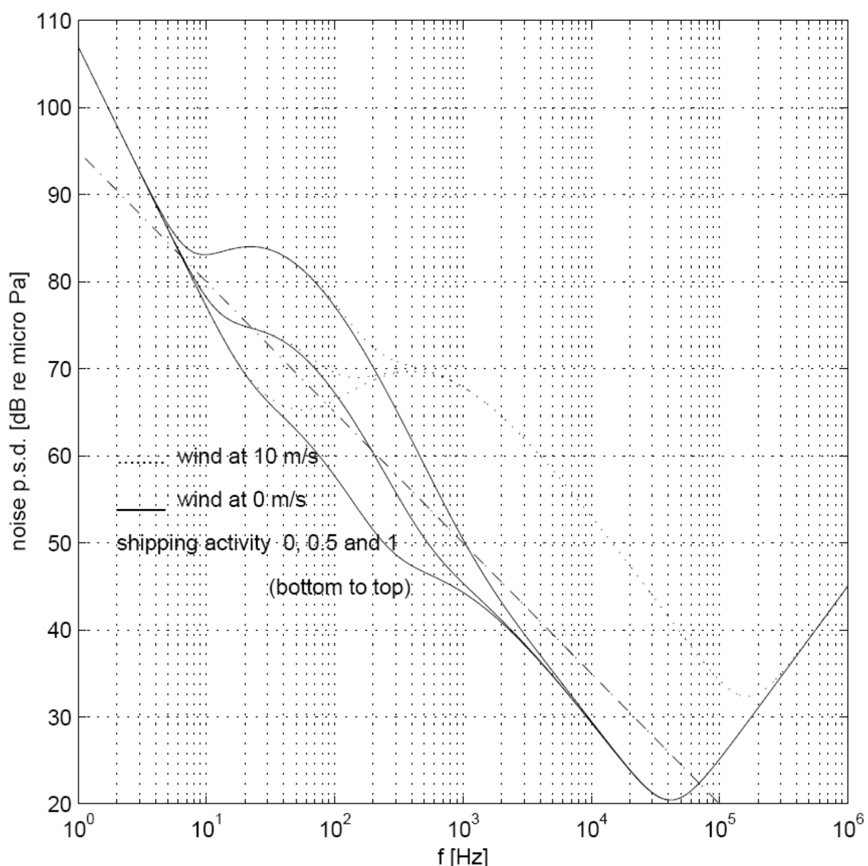




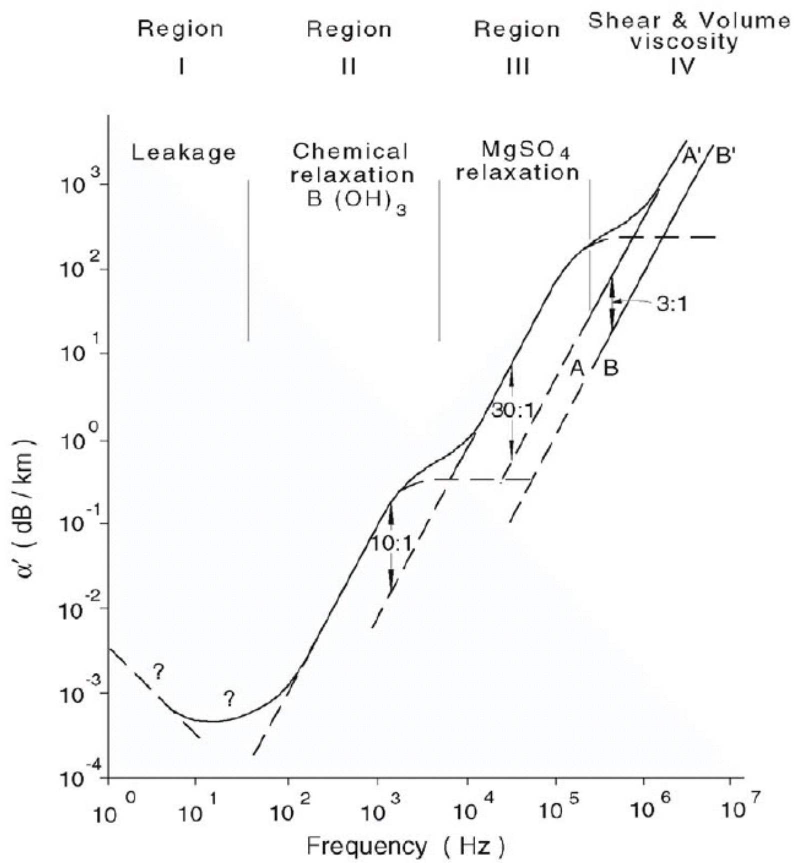
Figures from J. Preisig

Ambient Noise and Attenuation

- Ambient noise
  - Passing ships, storms, breaking waves, seismic events, wildlife

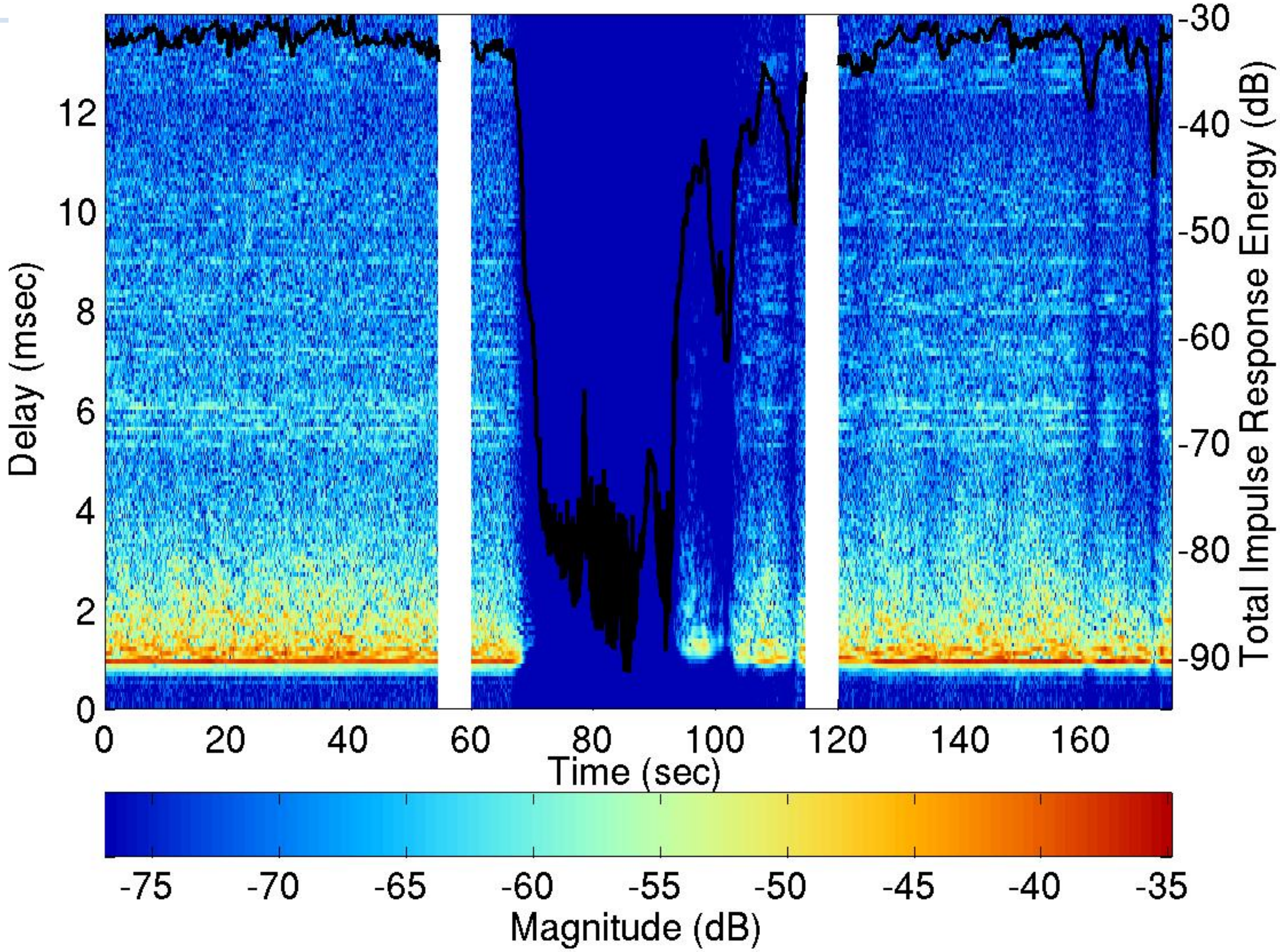


Stojanovic, WUWNeT'06



Schmidt, *Computational Ocean Acoustics*

Bubble Cloud Attenuation



Figures from J. Preisig

Path loss and Absorption

- Path Loss
  - Spherical Spreading ~ r^{-1}
  - Cylindrical Spreading ~ $r^{-0.5}$

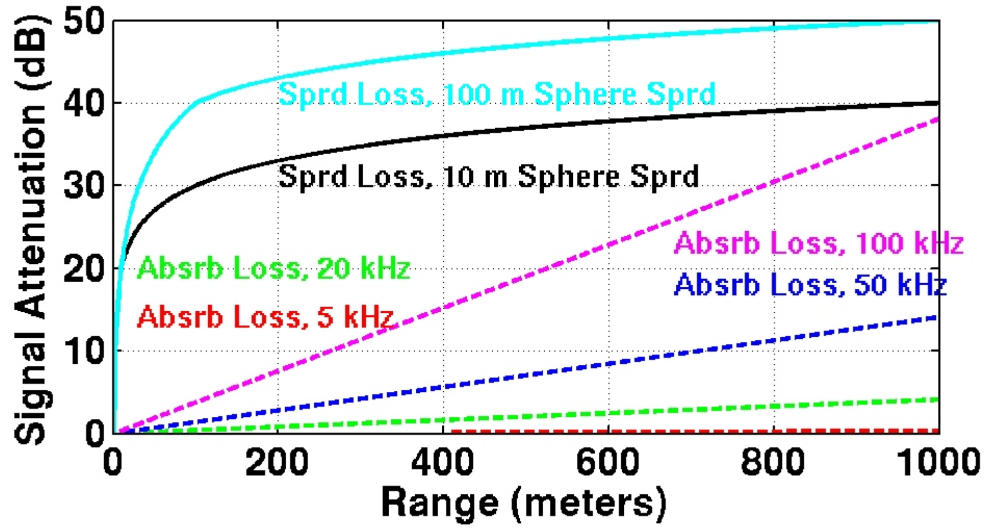


Figure from J. Preisig

- Absorption ~ $\alpha(f)^{-r}$
  - Thorp's formula (for sea water):

$$10\log\alpha(f) = 0.11\frac{f^2}{1+f^2} + 44\frac{f^2}{4100+f^2} + 0.000275 f^2 + 0.003 \quad \text{(dB/km)}$$

Long Range Bandwidth

$$SNR(f) = 171 + 10\log(P) - r\,\alpha(f) - 20\log(h/2) - 10\log(r - h/2) - 10\log\left(\int_{f-df/2}^{f+df/2} N(f)\,df\right) \text{ dB}$$

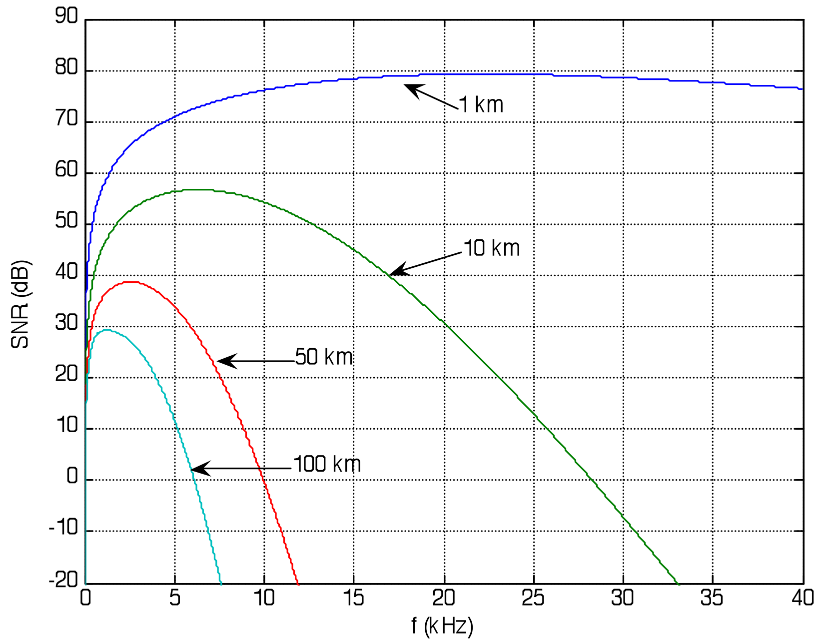


Figure courtesy of:
Costas Pelekanakis, Milica Stojanovic

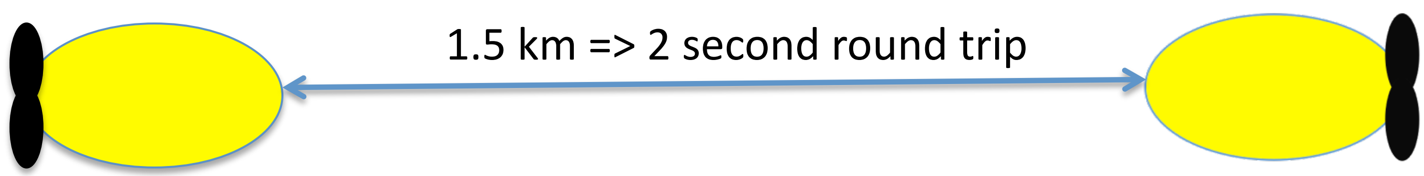
Source Power: P = 20 Watts

Water depth: h = 500 m

- Low modulation frequency
- System inherently wide-band
- Frequency curtain effect
  - Form of covert communications
  - Might help with network routing

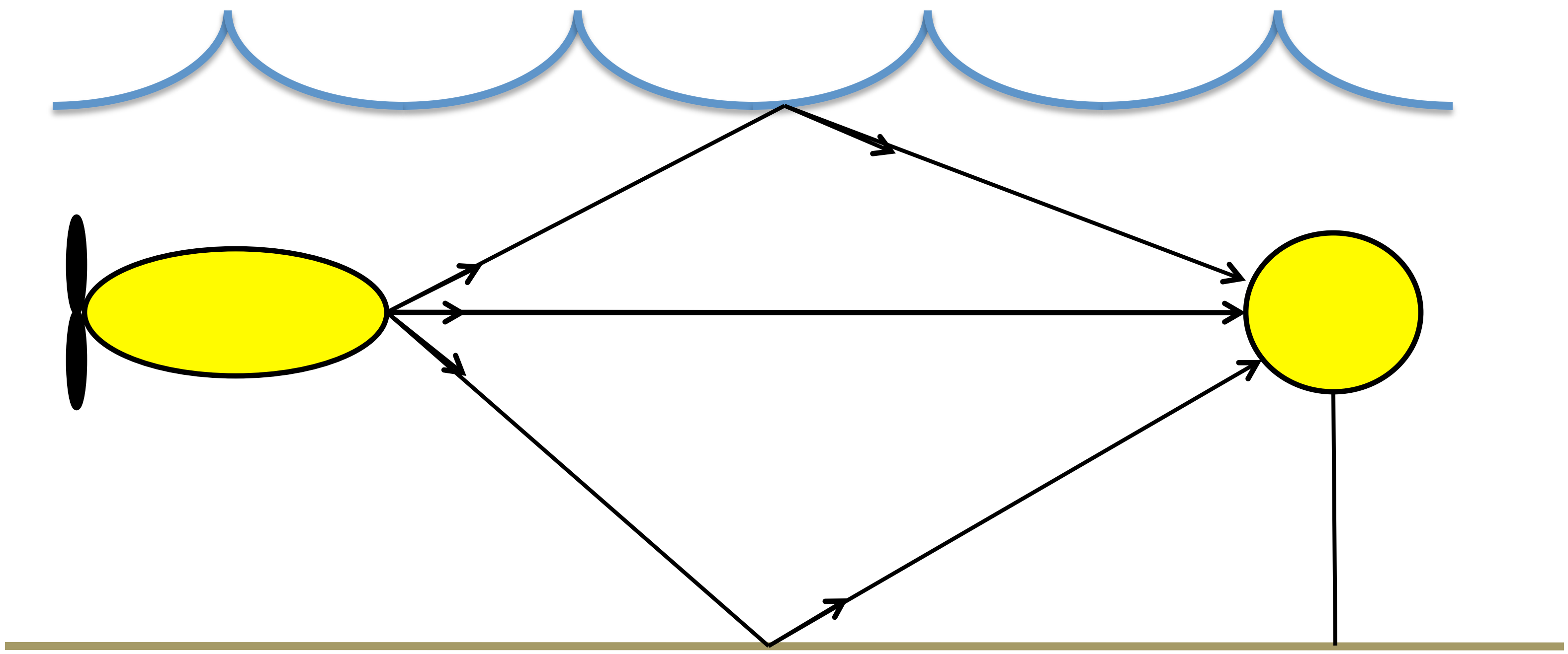
Latency and Power

- Propagation of sound slower than light
  - Feedback might take several second
  - Feedback must not be too time sensitive

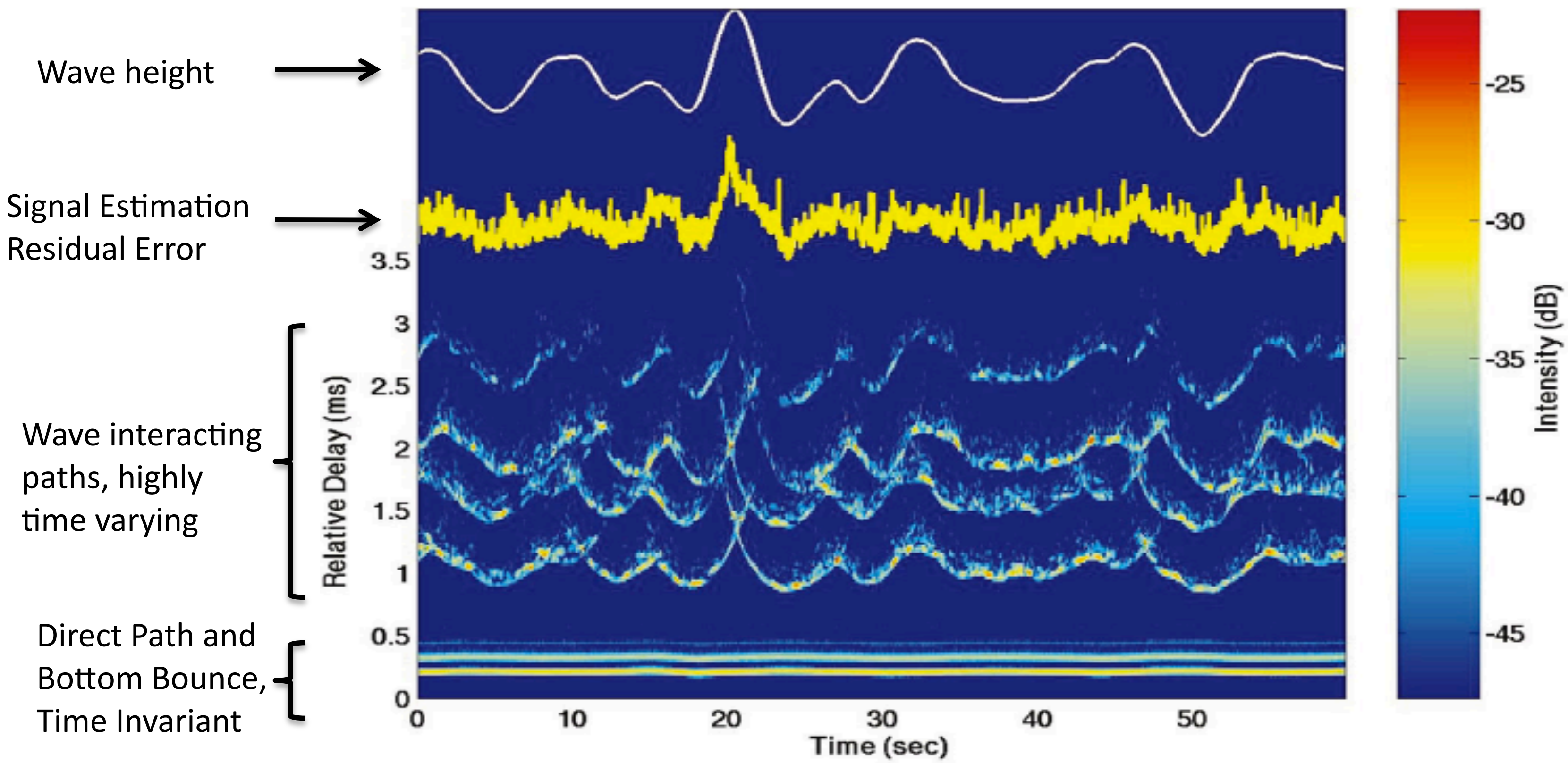


- Most underwater nodes battery powered
  - Communications Tx power (~10-100W)
  - Retransmissions costly

Shallow Water Multipath

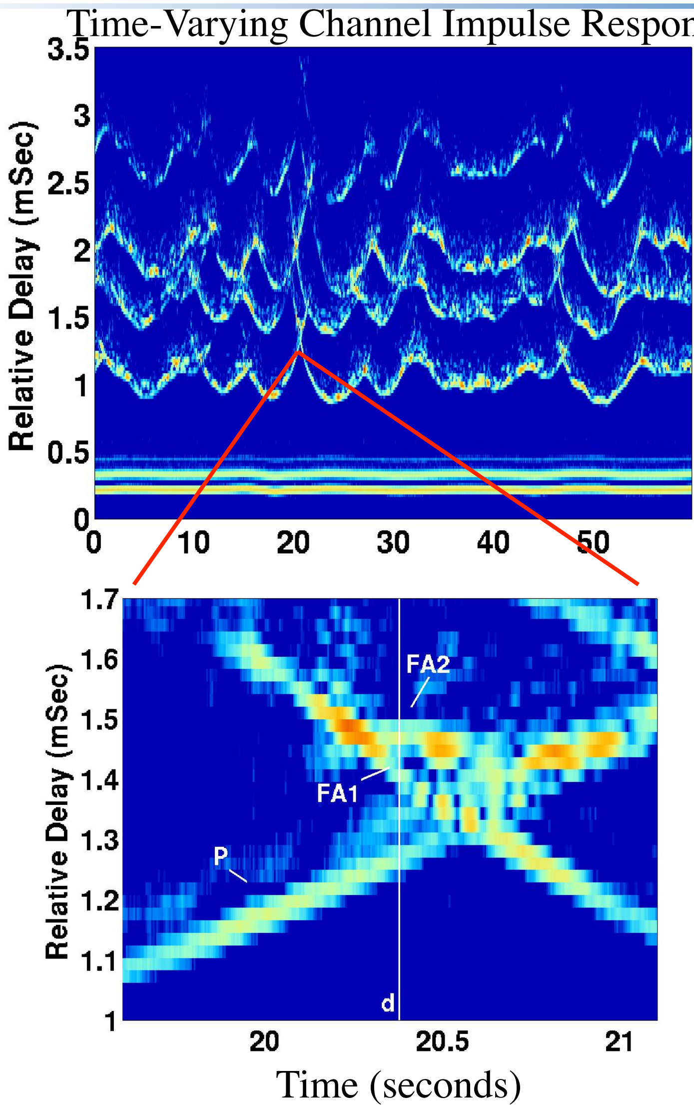


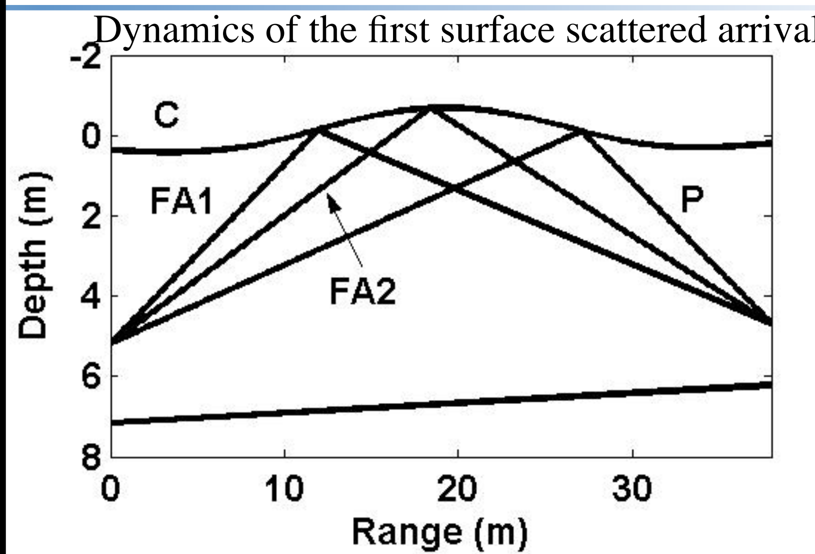
Time Varying Impulse Response

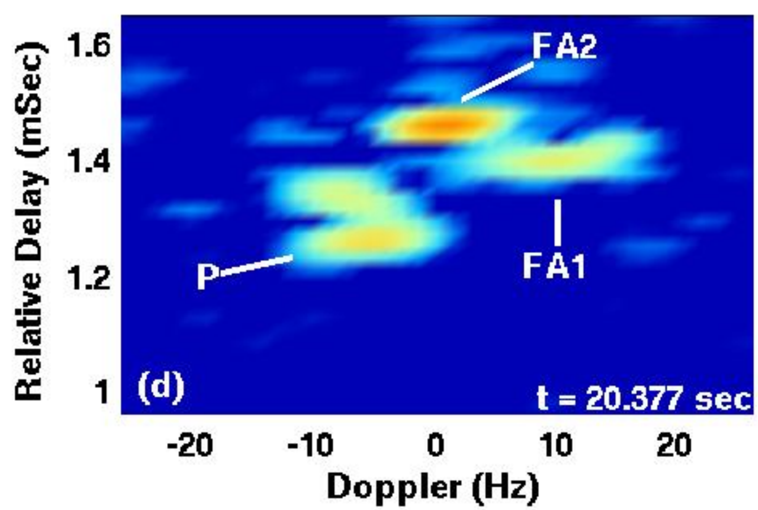


Wavefronts II Experiment from San Diego, CA
Preisig and Dean, 2004

Acoustic Focusing by Surface Waves







Preisig, 2006

Doppler Shifting / Spreading

$$a = v/c$$

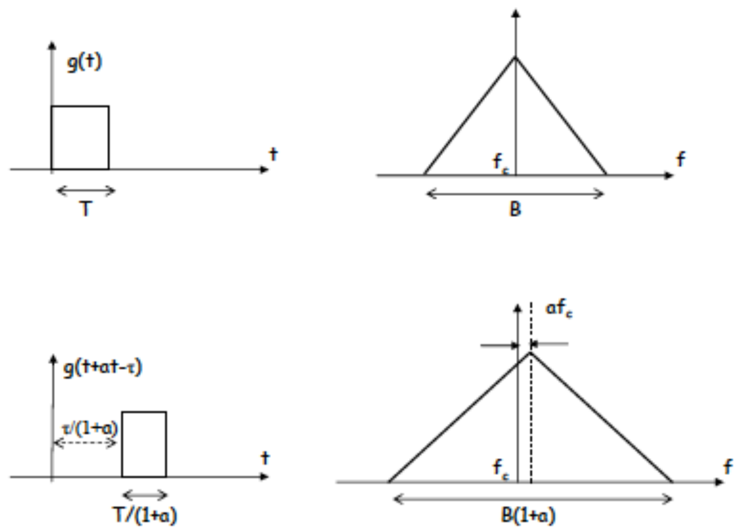


Fig. 8. Motion causes changes in the signal duration and frequency. The Doppler factor $a = v/c$ in an acoustic channel can be several orders of magnitude greater than in a radio channel.

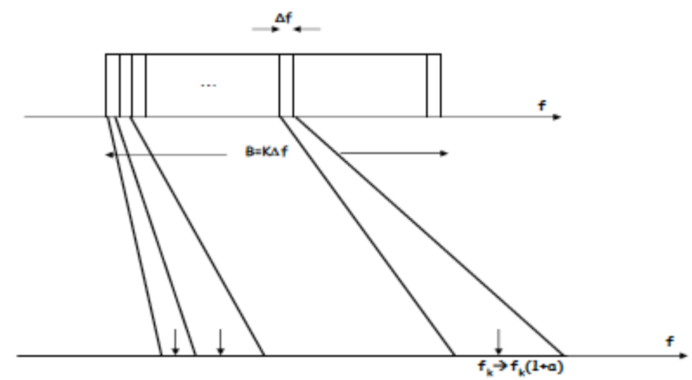


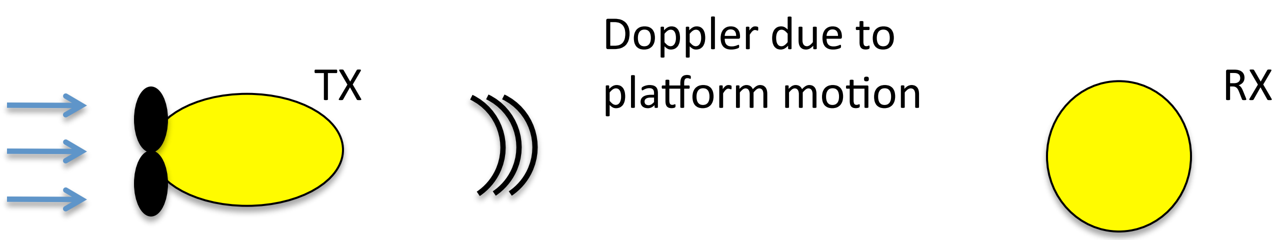
Fig. 9. Motion-induced Doppler shift is not uniform in a wideband system.

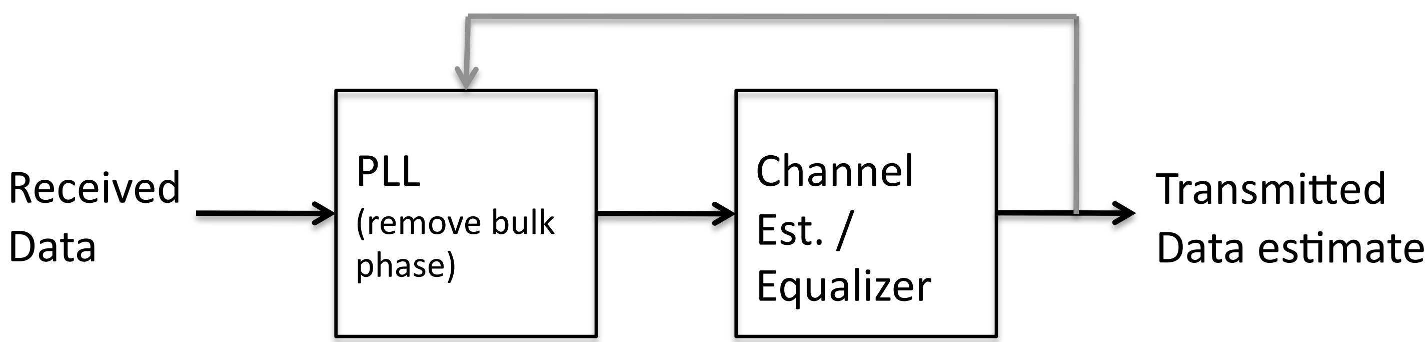
Stojanovic, 2008

Bulk Phase Removal

$$y_k(n) \approx d_k(n)H_k(n)e^{j\theta_k(n)} + z_k(n)$$

$$\theta_k(n+1) = \theta_k(n) + \Delta a(n) 2\pi f_k T'$$

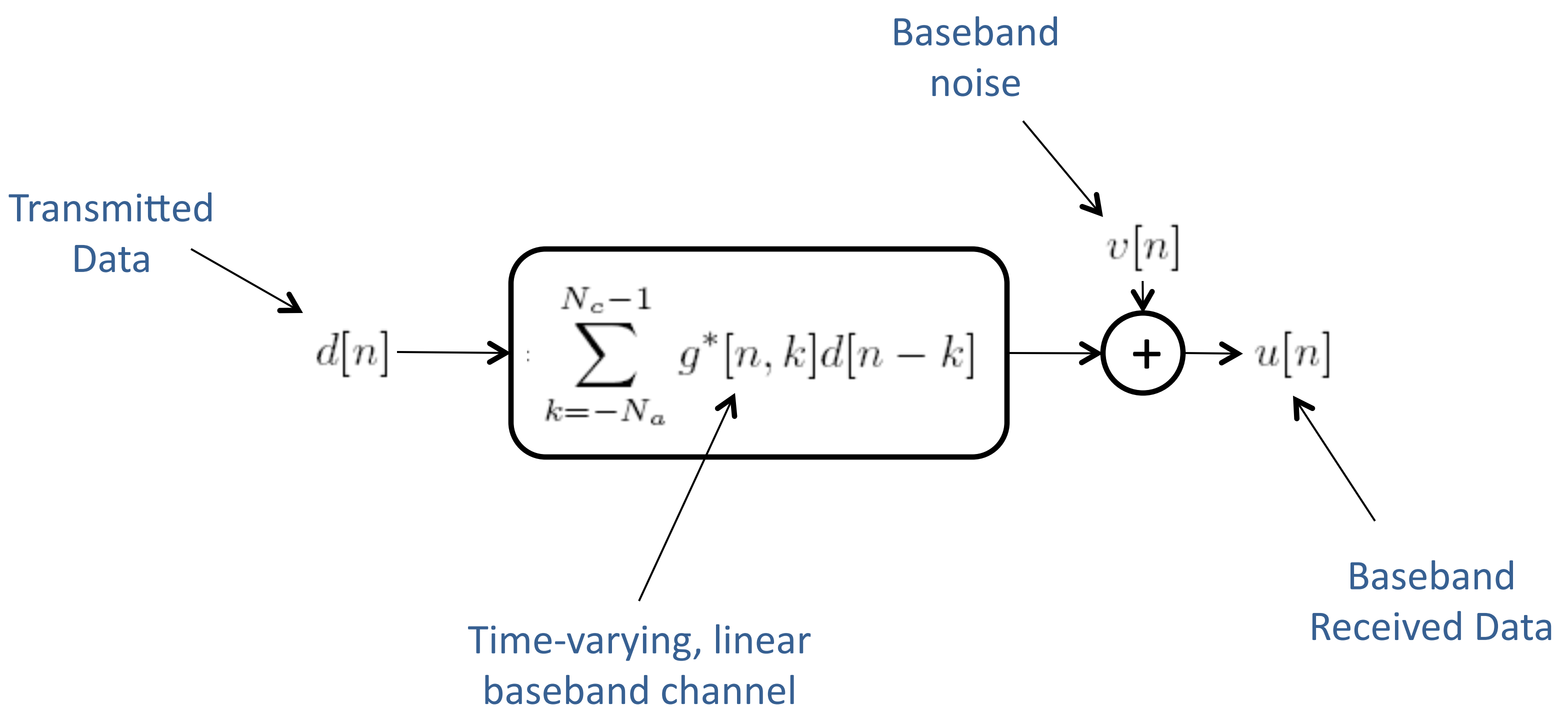




Multipath and Time Variability Implications

- Channel tracking and quality prediction is vital
  - Equalizer necessary and complex
- Coding and interleaving
- Network message routing can be challenging

Channel Model



Matrix-vector Form:

Split Channel Convolution Matrix

$$\mathbf{u}[n] = \mathbf{G}[n]\mathbf{d}[n] + \mathbf{v}[n] = \underbrace{\mathbf{G_{fb}}[n]\mathbf{d_{fb}}[n] + \mathbf{G_0}[n]\mathbf{d_0}[n]} + \mathbf{v}[n]$$

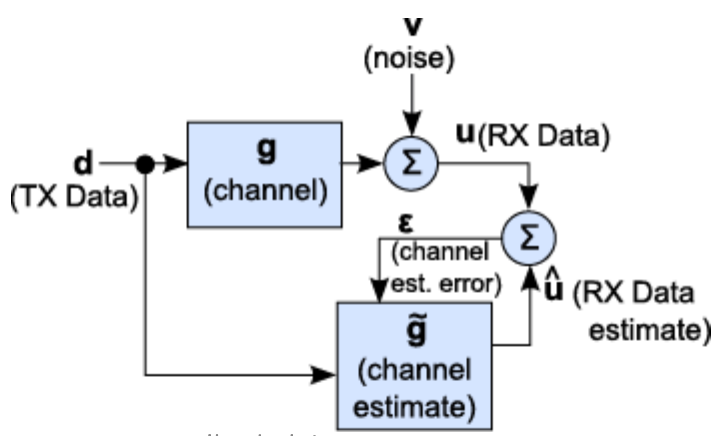
Time-domain Channel Estimation

LMMSE Optimization: $$\hat{\mathbf{g}}_{\text{opt}}[n] = \arg\min_{\mathbf{g}'} \mathrm{E}\{|\mathbf{g}'^H[n]\mathbf{d}[n] - u[n]|^2\}$$

Solution: $$\hat{\mathbf{g}}_{\text{opt}} = \mathbf{R_d}^{-1}\mathbf{r_{du}}$$

$$\mathbf{R_d} = \mathrm{E}\{\mathbf{d}[n]\mathbf{d}^H[n]\}$$
$$\mathbf{r_{du}} = \mathrm{E}\{\mathbf{d}[n]u^*[n]\}$$

Block Diagram:



Time-domain Equalization

TX Data bit (linear) estimator: $\hat{d}[n] = \mathbf{h}^H[n]\mathbf{z}[n]$

Vector of RX data and TX data estimates

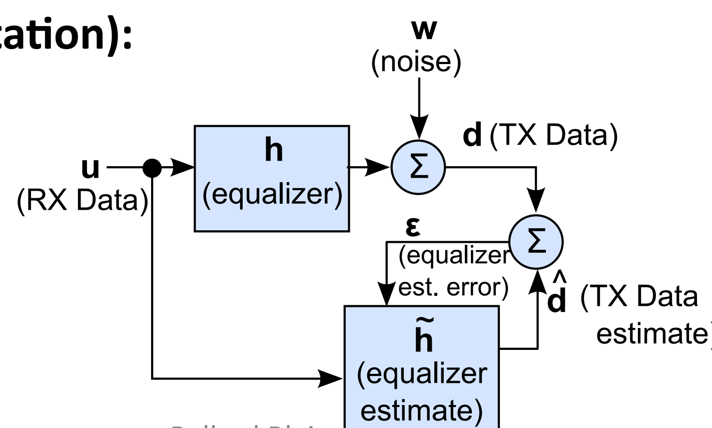
LMMSE Optimization: $\hat{\mathbf{h}}_{\text{opt}} = \underset{\mathbf{h}'}{\arg\min} \, \text{E}\{|\mathbf{h}^H\mathbf{z} - d|^2\}$

Solution: $\hat{\mathbf{h}}_{\text{opt}}[n] = \mathbf{R_z}^{-1}[n]\mathbf{r_{zd}}[n]$

$$\mathbf{R_z}[n] = \text{E}\{\mathbf{z}\mathbf{z}^H\}$$

$$\mathbf{r_{zd}} = \text{E}\{\mathbf{z}d^*\}$$

Block Diagram (direct adaptation):



Decision Feedback Equalizer (DFE)

- Problem Setup: $\hat{\mathbf{h}}_{\text{opt}} = \arg\min_{\mathbf{h}'} \mathrm{E}\{|\mathbf{h}^H \mathbf{z} - d|^2\}$
- Estimate using RX data and TX data estimates

$$\mathbf{z}[n] \quad = \quad [u[n - Lc + 1] \ \ldots \ u[n] \ \ldots \ u[n + L_a], \ \hat{d}[n-1] \ \ldots \ \hat{d}[n - L_{fb}]]^T$$

- DFE Eq: $\hat{d}[n] = \hat{\mathbf{h}}_{\text{opt}}^H[n]\mathbf{z}[n] = \mathbf{h}_{\mathbf{ff}}^H \mathbf{u}[\mathbf{n}] + \mathbf{h}_{\mathbf{fb}}^H \hat{\mathbf{d}}_{\mathbf{fb}}$

Solution to Weiner-Hopf Eq.

$$\begin{aligned} \hat{\mathbf{h}}_{\text{opt}}[n] &= \mathbf{R_z}^{-1}[n]\mathbf{r_{zd}}[n] \\ \mathbf{R_z}[n] &= \mathrm{E}\{\mathbf{z}\mathbf{z}^H\} \\ \mathbf{r_{zd}} &= \mathrm{E}\{\mathbf{z}d^*\} \end{aligned}$$

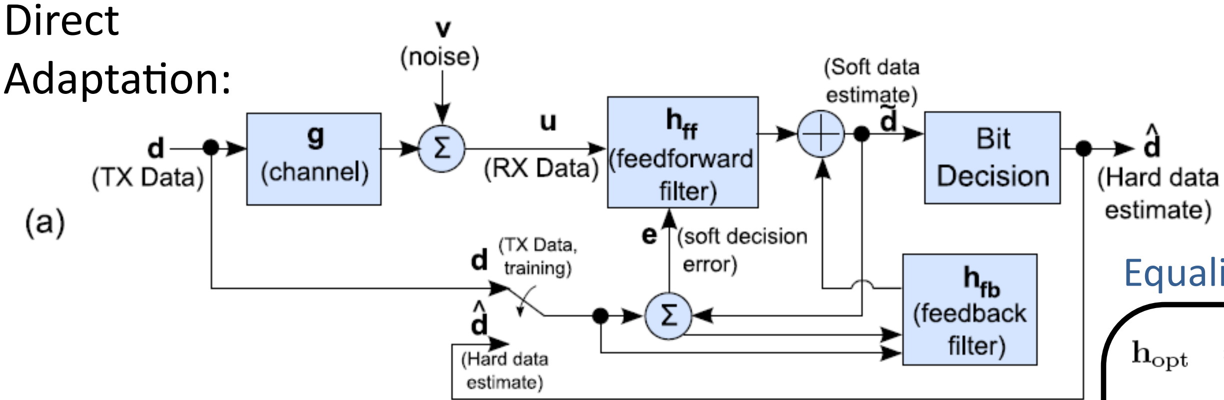
MMSE Sol. Using Channel Model

$$\begin{aligned} \mathbf{h_{ff}} &= [\mathbf{G}_0\mathbf{G}_0 + \mathbf{R_v}]^{-1}\mathbf{g}_0 \\ \mathbf{h_{fb}} &= -\mathbf{G_{fb}}\mathbf{h_{ff}} \end{aligned}$$

- Two Parts:
  - (Linear) feed-forward filter (of RX data)
  - (Linear) feedback filter (of data estimates)

DFE Strategies

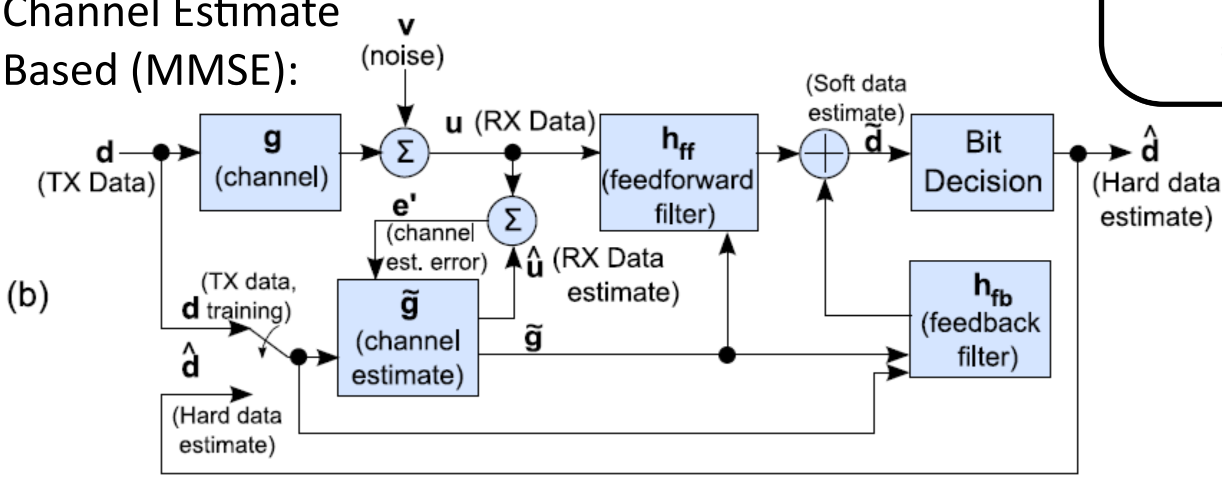
Direct Adaptation:



Equalizer Tap Solution:

$$
\begin{aligned}
\mathbf{h}_{\text{opt}} &= \begin{bmatrix} \mathbf{h}_{\mathbf{ff}} \\ \mathbf{h}_{\mathbf{fb}} \end{bmatrix} \\
&= \left(\mathrm{E}[\mathbf{z}\mathbf{z}^H]\right)^{-1} \mathrm{E}[\mathbf{z}d^*] \\
&= \left(\mathrm{E}\left[\begin{bmatrix} \mathbf{u} \\ \mathbf{d}_{\mathbf{fb}} \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ \mathbf{d}_{\mathbf{fb}} \end{bmatrix}^H\right]\right)^{-1} \mathrm{E}\left[\begin{bmatrix} \mathbf{u} \\ \mathbf{d}_{\mathbf{fb}} \end{bmatrix} d^*\right] \\
&= \begin{bmatrix} [\mathbf{G}_0\mathbf{G}_0 + \mathbf{R}_{\mathbf{v}}]^{-1}\mathbf{g}_0 \\ -\mathbf{G}_{\mathbf{fb}}\mathbf{h}_{\mathbf{ff}} \end{bmatrix}
\end{aligned}
$$

Channel Estimate Based (MMSE):

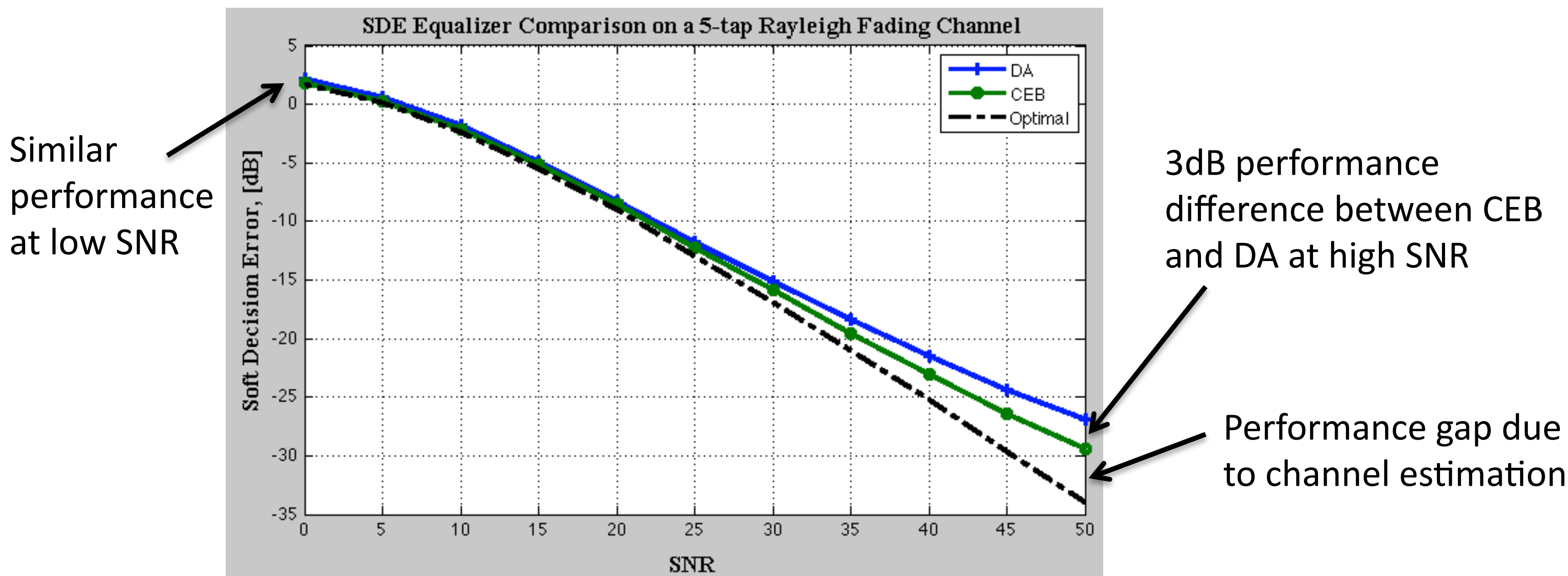


Question:

Why is the performance of a channel estimate based equalizer different than a direct adaptation equalizer?

Comparison between DA and CEB

- In the past, CEB methods empirically shown to have lower mean squared error at high SNR
- Reasons for difference varied:
  - Condition number of correlation matrix
  - Num. of samples required to get good estimate



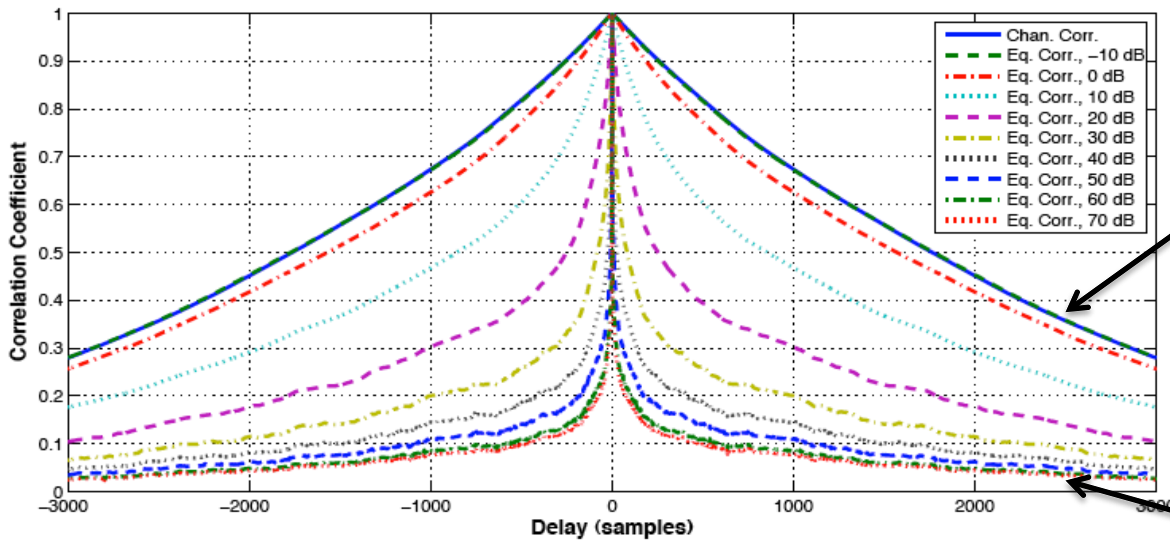
Comparison between DA and CEB

- Our analysis shows the answer is:
  Longer *corr. time for channel coefficients than MMSE equalizer coefficients at high SNR*

- Will examine low SNR and high SNR regimes
  - Use simulation to show transition of correlation time for the equalizer coefficients from low to high is smooth

Correlation over SNR – 1-tap

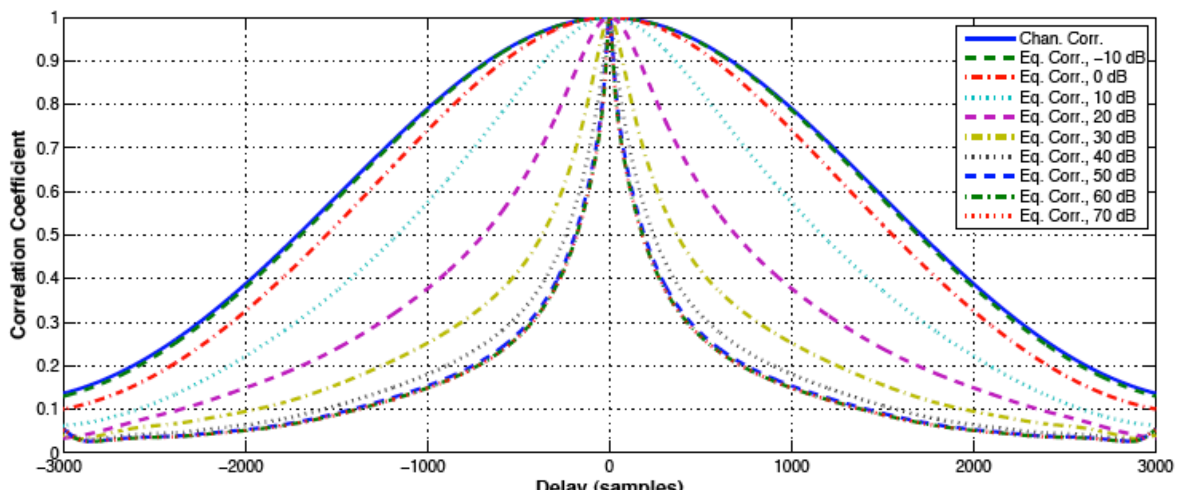
AR(1) model



Channel and Equalizer Coeff. Correlation the Same at low SNR

Equalizer Coeff. Correlation reduces as SNR increases

Gaussian model



Take-home Message

- Channel impulse-response taps have longer correlation time than MMSE equalizer taps
  - DA has greater MSE than CEB
- For time-invariant statistics, CEB and DA algorithms have similar performance
  - Low-SNR regime (assuming stationary noise)
  - Underwater channel operates in low SNR regime (<35dB)

Question:

How does the structure of the observed noise correlation matrix affect equalization performance?

Recall DFE Equations (again)

- DFE Eq: $\hat{d}[n] = \hat{\mathbf{h}}_{\text{opt}}^{H}[n]\mathbf{z}[n] = \mathbf{h}_{\text{ff}}^{H}\mathbf{u}[\mathbf{n}] + \mathbf{h}_{\text{fb}}^{H}\hat{\mathbf{d}}_{\text{fb}}$

Solution to Weiner-Hopf Eq.

$$\begin{aligned} \hat{\mathbf{h}}_{\text{opt}}[n] &= \mathbf{R_z}^{-1}[n]\mathbf{r_{zd}}[n] \\ \mathbf{R_z}[n] &= \mathrm{E}\{\mathbf{z}\mathbf{z}^H\} \\ \mathbf{r_{zd}} &= \mathrm{E}\{\mathbf{z}d^*\} \end{aligned}$$

MMSE Sol. Using Channel Model

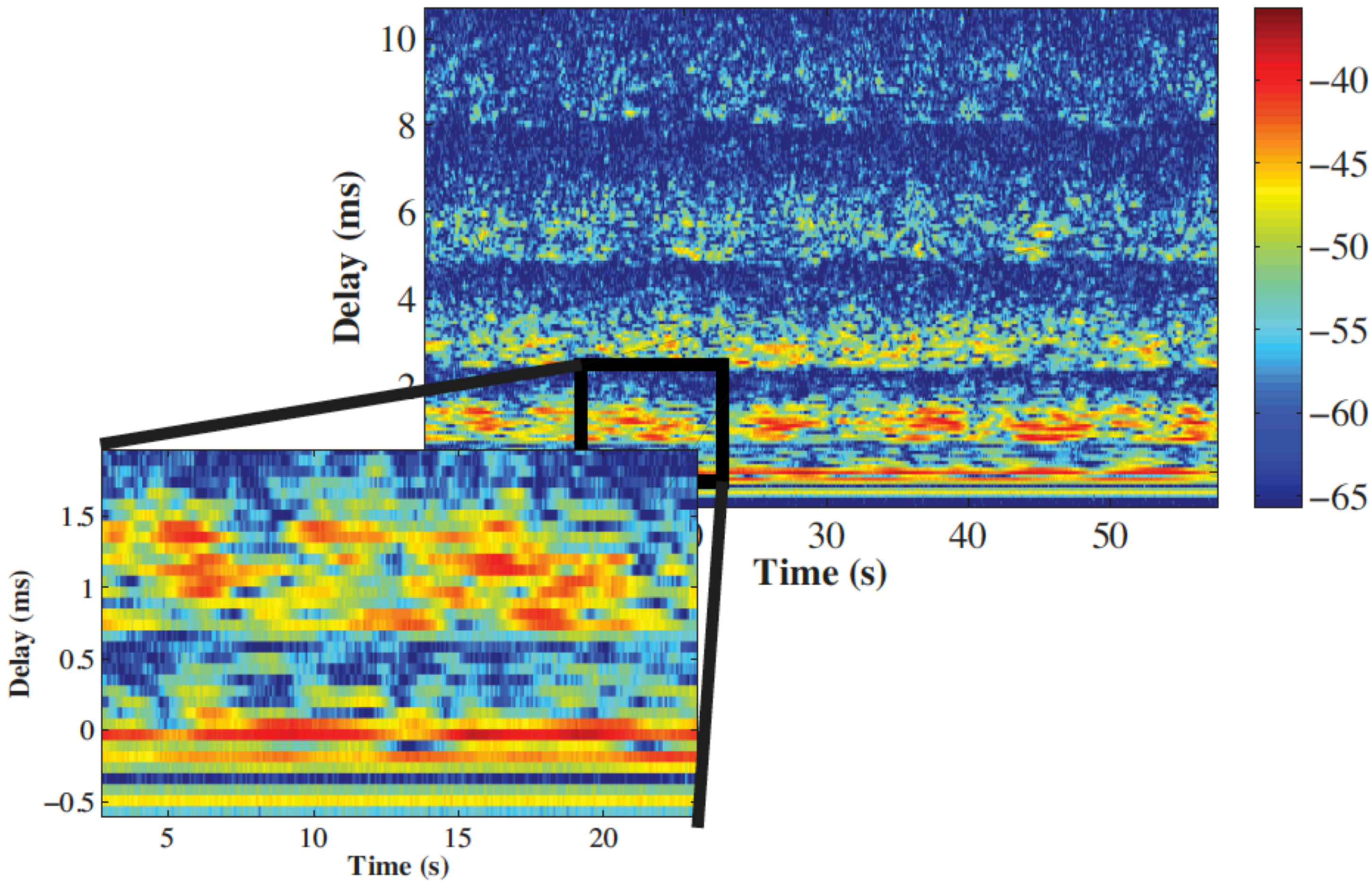
$$\begin{aligned} \mathbf{h}_{\text{ff}} &= [\mathbf{G}_0\mathbf{G}_0 + \mathbf{R_v}]^{-1}\mathbf{g}_0 \\ \mathbf{h}_{\text{fb}} &= -\mathbf{G}_{\text{fb}}\mathbf{h}_{\text{ff}} \end{aligned}$$

- Vector of data RX data and TX data est.

$$\mathbf{z}[n] = \left[u[n-Lc+1] \ \dots \ u[n] \ \dots \ u[n+L_a], \ \hat{d}[n-1] \ \dots \ \hat{d}[n-L_{fb}]\right]^T$$

- Assumed noise covariance form: $\mathbf{R_v} = \rho\mathbf{I}$

Channel Correlations



Updated Equalizer Equations

- Channel Estimation Model: $\mathbf{G}[n] = \hat{\mathbf{G}}[n] + \Gamma[n]$
- Effective Noise: $\mathbf{w}[n] = \Gamma[n]\mathbf{d}[n] + \mathbf{v}[n]$

- New DFE Eq. Equations:

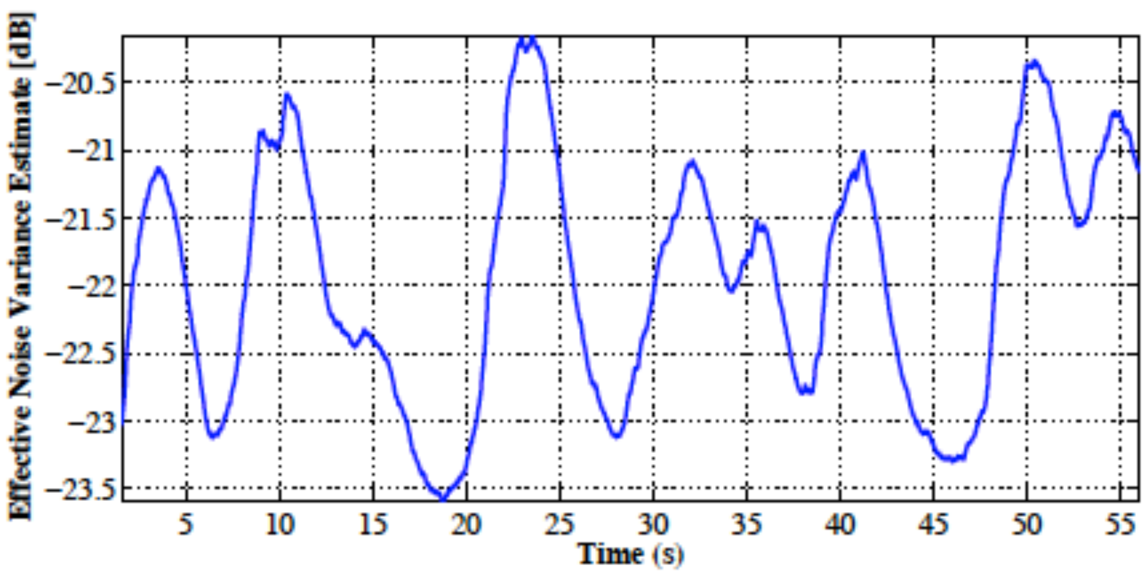
$$\begin{aligned} \mathbf{h}_{\mathrm{ff}}[n] &= (\hat{\mathbf{G}}_0[n]\hat{\mathbf{G}}_0^H[n] + \mathbf{R}_\Gamma[n] + \sigma_d^{-2}\mathbf{R}_\mathbf{v}[n])^{-1}\mathbf{g}_0 \\ \mathbf{h}_{\mathrm{fb}}[n] &= -\hat{\mathbf{G}}_{\mathrm{fb}}{}^H[n]\mathbf{h}_{\mathrm{ff}}[n] \end{aligned} \quad (12)$$

- Effective Noise Term:

$$\mathbf{R}_0 = \mathbf{R}_\Gamma[n] + \sigma_d^{-2}\mathbf{R}_\mathbf{v}[n]$$

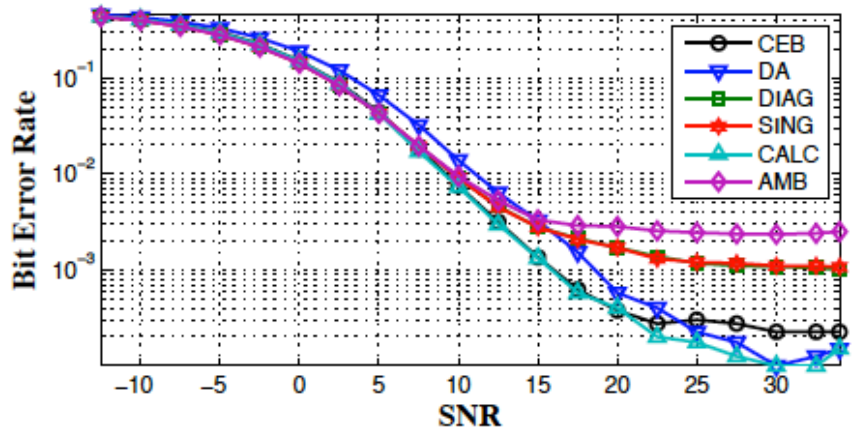
Effective Noise variance from data

- SPACE08 Experiment
  - Estimate of top-left element of R_0

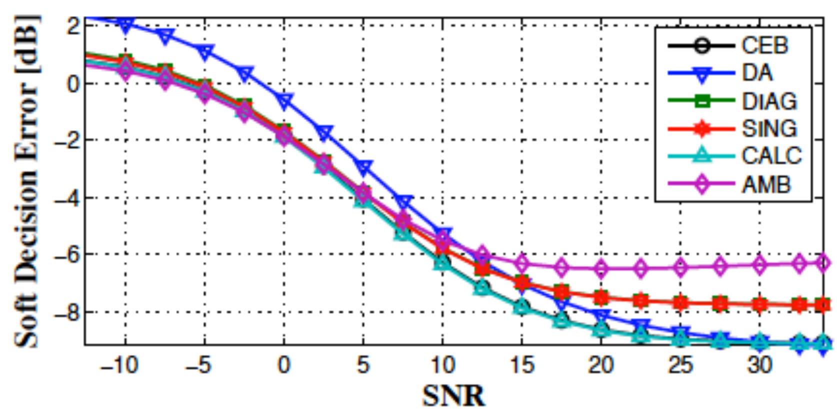


Comparison of Algorithms

- SPACE08 Data (training mode)



(a) Bit error rate (BER) results



(b) Soft decision error (SDE) results

Take-home message

- Diagonal noise correlation matrix is not sufficient for the underwater channel
- Need to track noise variance throughout packet
- Noise statistics are slowly varying, so can assume matrix is Toeplitz
  - Reduces algorithmic complexity

Direct Adaptation Equalization

$$
\begin{aligned}
\mathbf{h}_{\text{opt}} &= \begin{bmatrix} \mathbf{h}_{\text{ff}} \\ \mathbf{h}_{\text{fb}} \end{bmatrix} \\
&= \left(\mathrm{E}[\mathbf{z}\mathbf{z}^H]\right)^{-1} \mathrm{E}[\mathbf{z}d^*] \\
&= \left(\mathrm{E}\left[\begin{bmatrix} \mathbf{u} \\ \mathbf{d}_{\text{fb}} \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ \mathbf{d}_{\text{fb}} \end{bmatrix}^H\right]\right)^{-1} \mathrm{E}\left[\begin{bmatrix} \mathbf{u} \\ \mathbf{d}_{\text{fb}} \end{bmatrix} d^*\right] \\
&= \begin{bmatrix} [\mathbf{G}_0\mathbf{G}_0 + \mathbf{R}_{\mathbf{v}}]^{-1}\mathbf{g}_0 \\ -\mathbf{G}_{\text{fb}}\mathbf{h}_{\text{ff}} \end{bmatrix}
\end{aligned}
$$

Model Assumptions

- Does not require (or use) side information
- More computationally efficient
  - – $O(N^2)$ vs $O(N^3)$

Future Directions and Ideas

- Methods to reduce degrees of freedom to be estimated
  - Sparsity (very active area right now)
  - Physical Constraints

- Communication systems do not exist in a vacuum underwater
  - Usually on well instrumented platforms
  - How can additional information be used to improve communication?

Conclusions

- Research in underwater communications is still necessary and active
- The underwater channel is challenging

- Equalization
  – Bulk phase removal through PLL
  – DA equalization deserves another look
  – Cannot assume diagonal noise correlation matrix

Thanks!

Thanks to Prof. John Buck for inviting me today

For their time and comments:

- Jim Preisig
- Milica Stojanovic

Project funded by:

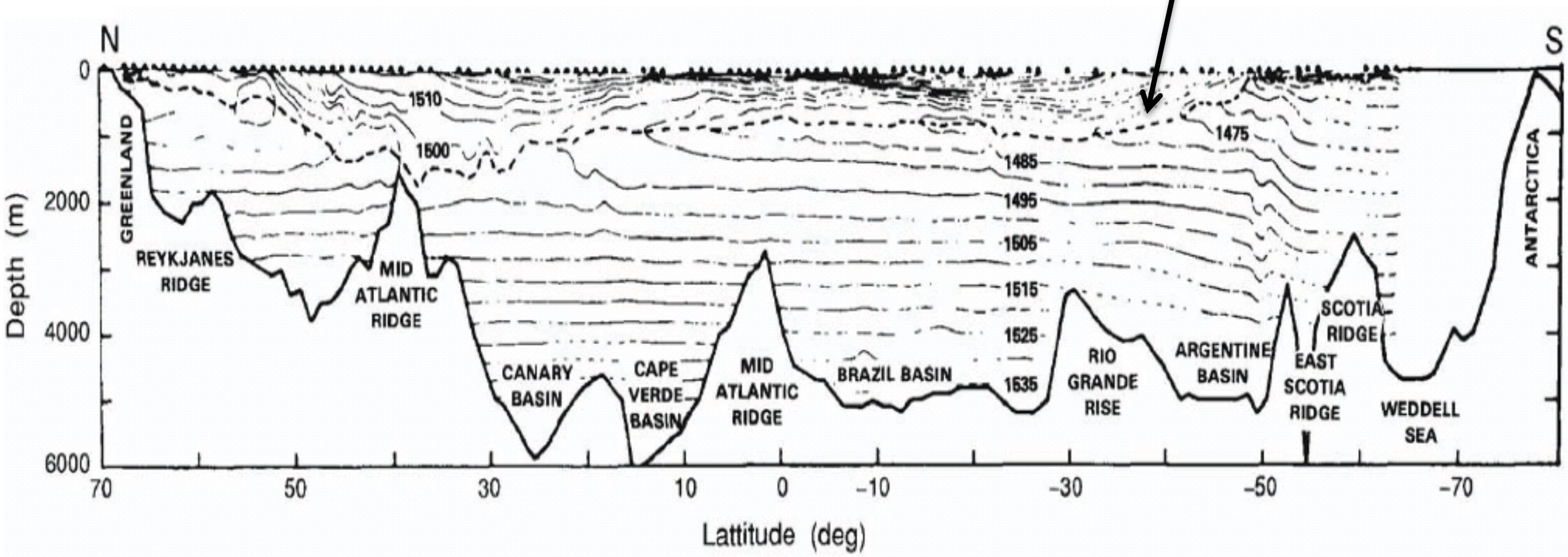
- The Office of Naval Research

Questions?

Backup Slides

Global Ocean Profile

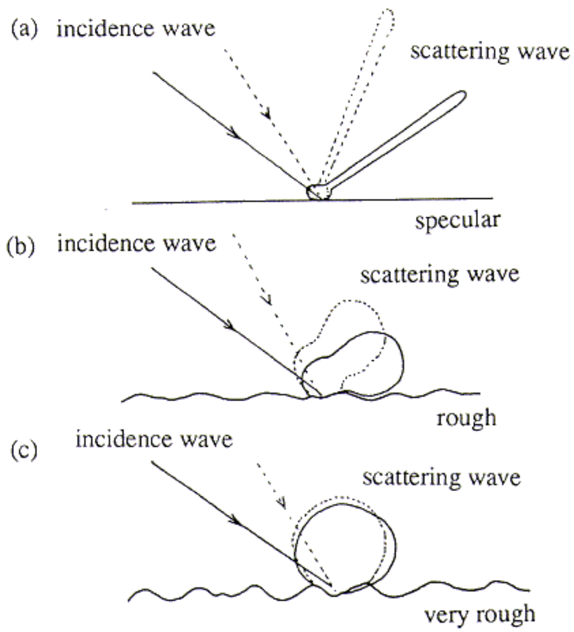
SOFAR Channel

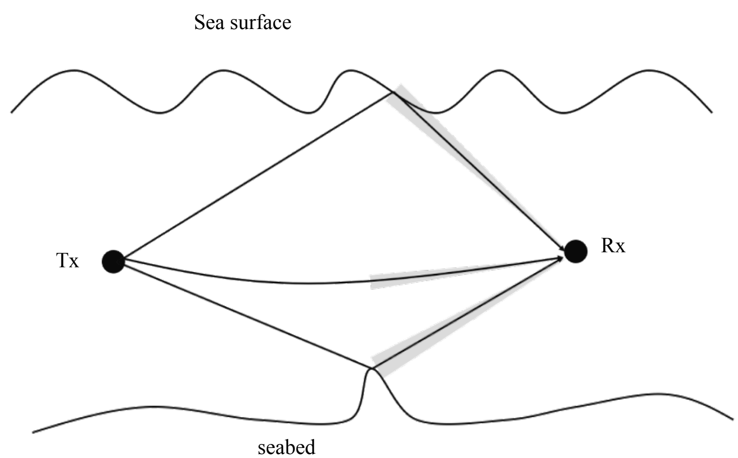


Schmidt, *Computational Ocean Acoustics*

Multipath

- Micro-multipath due to rough surfaces
- Macro-multipath due to environment



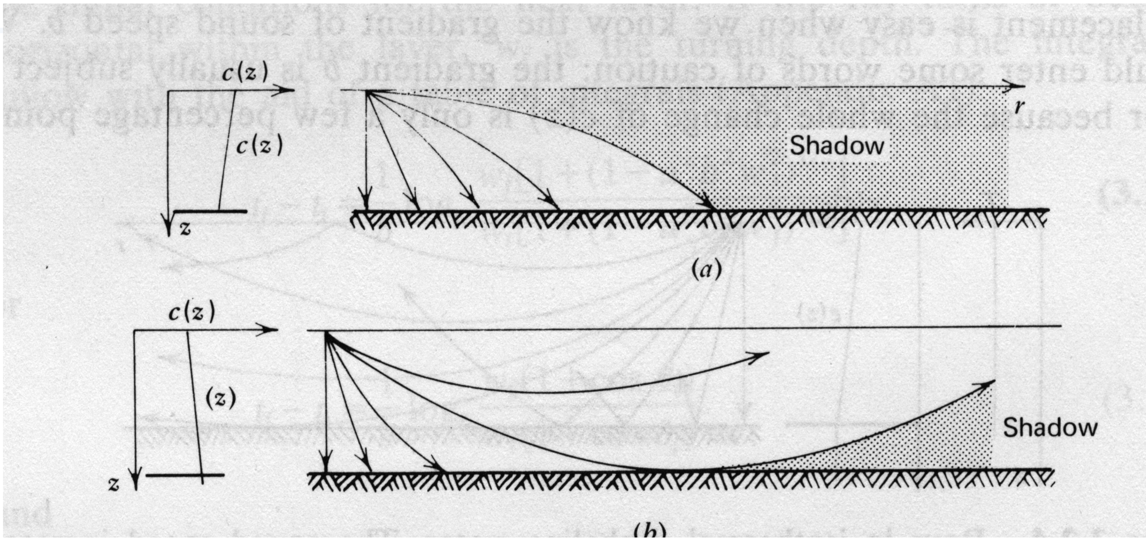


Speed of Sound Implications

- Vertical sound speed profile impacts
  - the characteristics of the impulse response
  - the amount and importance of surface scattering
  - the amount of bottom interaction and loss
  - the location and level of shadow zones

- Horizontal Speed of Sound impacts
  - Nonlinearities in channel response

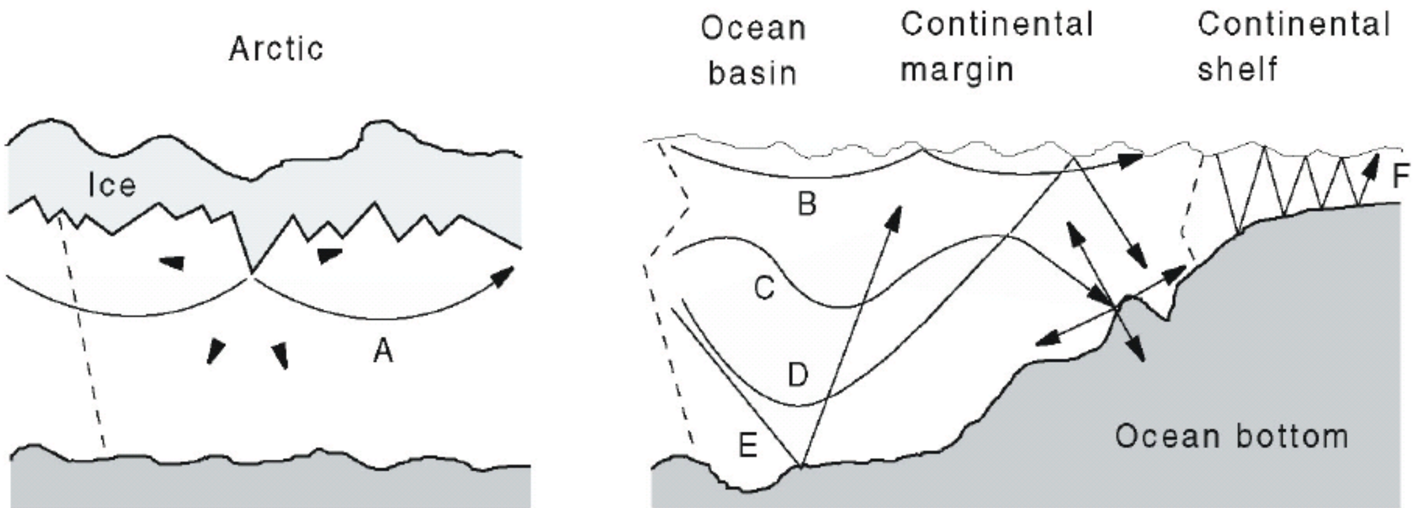
Shadow Zones



Clay and Medwin, "Acoustical Oceanography"

- **Sometimes there is no direct path (unscattered) propagation between two points. All paths are either surface or bottom reflected or there are no paths.**
- **Problem with communications between two bottom mounted instruments in upwardly refracting environment (cold weather shallow water, deep water).**
- **Problem with communications between two points close to the surface in a downwardly refracting environment (warm weather shallow water and deep water).**

Propagation Paths



A. Arctic
B. Surface duct
C. Deep sound channel
D. Convergence zone
E. Bottom bounce
F. Shallow water

Schmidt, *Computational Ocean Acoustics*

Assumptions

- Unit variance, white transmit data

$$\mathrm{E}\{\mathbf{d}[n]\mathbf{d}^H[n]\} = \mathbf{I}$$

- TX data and obs. noise are uncorrelated

$$\mathrm{E}\{\mathbf{v}[n]\mathbf{d}^H[m]\} = \mathbf{0}$$

  – Obs. Noise variance:

$$\mathbf{R}_\mathbf{v} = \mathrm{E}\{\mathbf{v}[n]\mathbf{v}^H[n]\}$$

- Perfect data estimation (for feedback)

$$\hat{\mathbf{d}} = \mathbf{d}$$

- Equalizer Length = Estimated Channel Length

$$N_a + N_c = L_a + L_c$$

- MMSE Equalizer Coefficients have form:

$$\begin{aligned}\mathbf{h}_{\mathbf{ff}} &= [\mathbf{G}_0\mathbf{G}_0^H + \mathbf{R}_\mathbf{v}]^{-1}\mathbf{G}\mathbf{s} \\ \mathbf{h}_{\mathbf{fb}} &= -\mathbf{G}_{fb}^H\mathbf{h}_{\mathbf{ff}}\end{aligned}$$

WSSUS AR channel model

- Simple channel model to analyze
- Similar to encountered situations

$$g[n+1] = \alpha g[n] + w[n]$$

$$R_{gg}[k] = \mathrm{E}\{g[n]g^*[n+k]\} = \begin{cases} \sigma_w^2 \left(\frac{(\alpha^*)^k}{1-|\alpha|^2} \right) & k \geq 0 \\ \sigma_w^2 \left(\frac{\alpha^{-k}}{1-|\alpha|^2} \right) & k < 0 \end{cases}$$

DFE: Notes

- Same expected squared estimate error

$$\mathrm{E}\{|\hat{e}_{\mathrm{dfe}}|^2\} = \boldsymbol{\Delta}\mathbf{G}\boldsymbol{\Delta}\mathbf{G}^H + \mathbf{R}_{\mathbf{v}}$$

- Strong error dependence on FB channel offset

- Cross term of separated offset is not necessarily diagonal $\boldsymbol{\Delta}\mathbf{G}_0\boldsymbol{\Delta}\mathbf{G}_0^H$

Acoustics Background

- Acoustic wave is compression wave traveling through water medium

Wave Equation for Pressure

$$\rho \nabla \cdot \left(\frac{1}{\rho} \nabla p \right) - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = 0 \,,$$

Wave Equation for Particle Velocity

$$\frac{1}{\rho} \nabla \left(\rho c^2 \, \nabla \cdot \mathbf{v} \right) - \frac{\partial^2 \mathbf{v}}{\partial t^2} = \mathbf{0} \,.$$

Time varying channel

- Time variation is due to:
  - Platform motion
  - Internal waves
  - Surface waves

- Effects of time variability
  - Doppler Shift $$f_d = f_c \frac{u}{c}$$
  - Time dilation/compression of the received signal

- Channel coherence times often << 1 second.

- Channel quality can vary in < 1 second.

Low SNR Regime

Update eqn. for feed-forward equalizer coefficients (AR model assumed):

$$\begin{aligned}
\mathbf{h}_{\mathbf{ff}}[n+1] &= (\mathbf{G}_0[n+1]\mathbf{G}_0^H[n+1] + \mathbf{R}_{\mathbf{v}})^{-1}(\mathbf{g}_0[n+1]) \\
&\approx \mathbf{R}_{\mathbf{v}}{}^{-1}(\alpha \mathbf{g}_0[n] + \mathbf{w}[n]) \\
&\approx \alpha \mathbf{h}_{\mathbf{ff}}[n] + \mathbf{R}_{\mathbf{v}}{}^{-1}\mathbf{w}[n]
\end{aligned}$$

Approximation:

$$\mathbf{R}_{\mathbf{v}} + \mathbf{G}[n]\mathbf{G}^H[n] \approx \mathbf{R}_{\mathbf{v}}$$

Has same correlation structure as channel coefficients

High SNR

$$\mathbf{h}_{\mathbf{ff}}[n+1] = (\mathbf{G}_0[n+1]\mathbf{G}_0^H[n+1] + \mathbf{R}_\mathbf{v})^{-1}(\mathbf{g}_0[n+1])$$

Approximation:

$$\mathbf{G}_0[n]\mathbf{G}_0^H[n] + \mathbf{R}_\mathbf{v} \approx \mathbf{G}_0[n]\mathbf{G}_0^H[n]$$

$$\rightarrow (\mathbf{G}_0[n]\mathbf{G}_0^H[n])\mathbf{h}_{\mathbf{ff}}[n] = \mathbf{g}_0[n]$$

Reduced Channel Convolution Matrix:

$$\mathbf{G}_0 = \begin{bmatrix} g_0^*[n-L+1] & 0 & 0 & \cdots & 0 \\ g_1^*[n-L+2] & g_0^*[n-L+2] & 0 & \cdots & 0 \\ g_2^*[n-L+3] & g_1^*[n-L+3] & g_0^*[n-L+3] & \cdots & 0 \\ \vdots & & & \ddots & \vdots \\ g_L^*[n] & g_{L-1}^*[n] & g_{L-2}^*[n] & \cdots & g_0^*[n] \end{bmatrix}$$

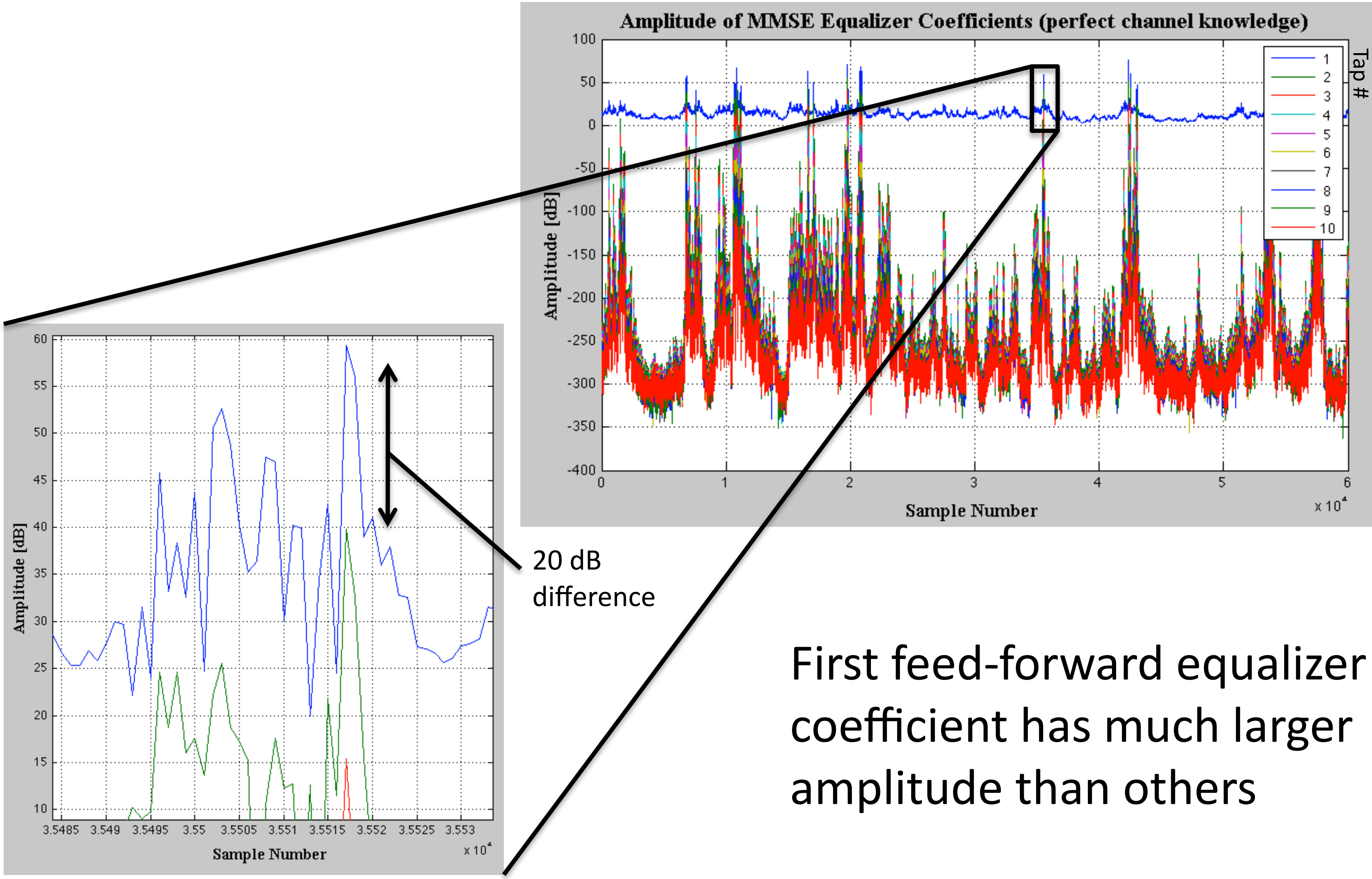
Matrix Product:

$$\mathbf{G}_0\mathbf{G}_0^H = \begin{bmatrix} |g_0|^2 & \cdots \\ g_0 g_1^* & \cdots \\ g_0 g_2^* & \cdots \\ \vdots & \vdots \\ g_0 g_L & \cdots \end{bmatrix}$$

Reduces to single tap:

$$\mathbf{h}[n] = \begin{bmatrix} 1/g_0 & 0 & 0 & \ldots & 0 \end{bmatrix}^T$$

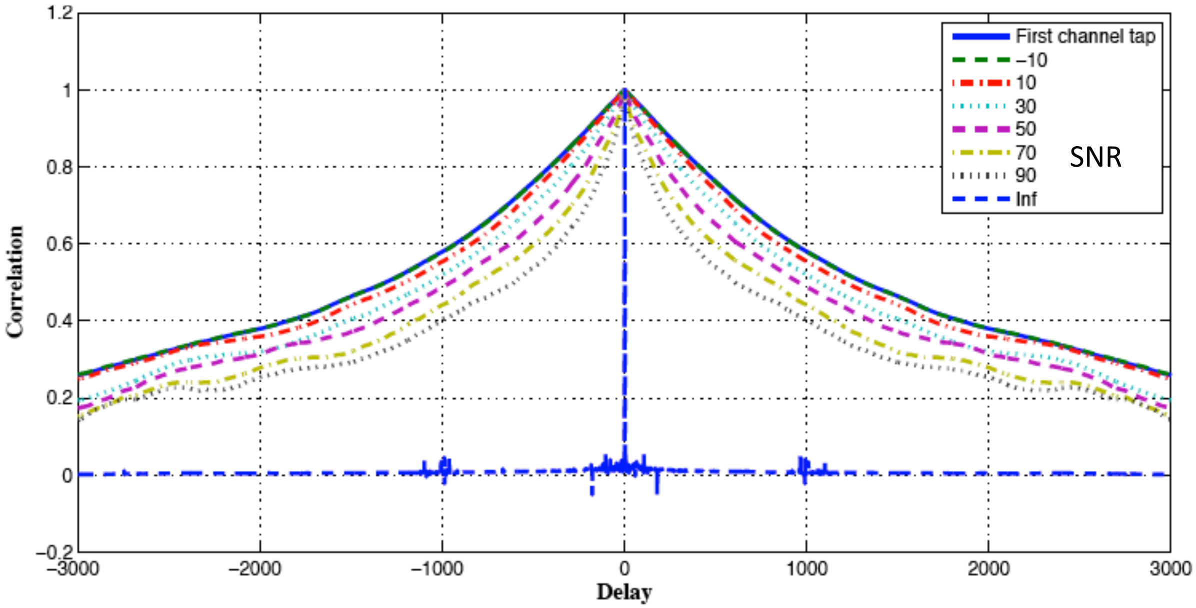
Amplitude of MMSE Eq. Coeff.

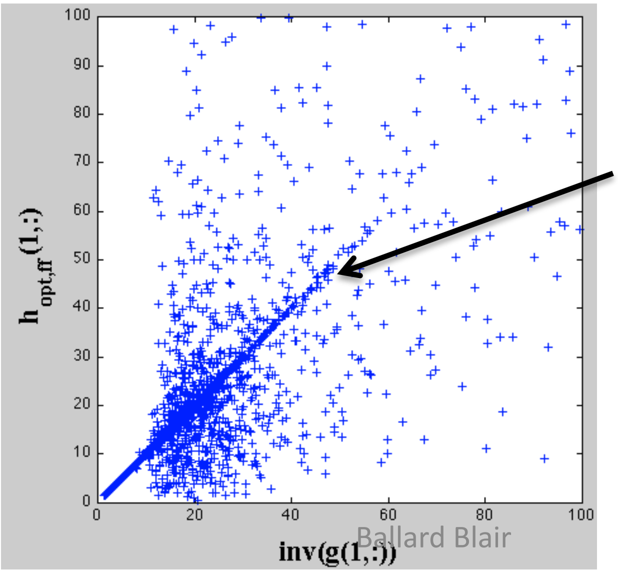


First feed-forward equalizer coefficient has much larger amplitude than others

Multi-tap correlation

Multi-tap AR(1) model





Strong linear correlation between inverse of first channel tap and first MMSE Eq. tap

Form of observed noise correlation

- Channel Estimation Model: $\mathbf{G}[n] = \hat{\mathbf{G}}[n] + \Gamma[n]$
- Data Model: $\mathbf{u}[n] = \mathbf{G}[n]\mathbf{d}[n] + \mathbf{v}[n] = \hat{\mathbf{G}}[n]\mathbf{d}[n] + \Gamma[n]\mathbf{d}[n] + \mathbf{v}[n]$

- Effective Noise: $\mathbf{w}[n] = \Gamma[n]\mathbf{d}[n] + \mathbf{v}[n]$
- Effective Noise Correlation:

$$\begin{aligned}
\mathbf{R_w}[n] &= \mathrm{E}\{\mathbf{w}[n]\mathbf{w}^H[n]\} \\
&= \mathrm{E}\{(\Gamma[n]\mathbf{d}[n] + \mathbf{v}[n])(\Gamma[n]\mathbf{d}[n] + \mathbf{v}[n])^H\} \\
&= \mathrm{E}\{\Gamma[n]\mathbf{d}[n]\mathbf{d}^H[n]\Gamma^H[n]\} + \mathbf{R_v}[n] \\
&= \mathrm{E}\{\Gamma[n]\mathrm{E}\{\mathbf{d}[n]\mathbf{d}^H[n]|\Gamma[n]\}\Gamma^H[n]\} + \mathbf{R_v} \\
\mathbf{R_w}[n] &= \sigma_d^2\mathbf{R}_\Gamma[n] + \mathbf{R_v}[n] \qquad (11)
\end{aligned}$$

Algorithm to estimate effective noise

- Calculate estimate of the effective noise:

$$\hat{\mathbf{e}}[n] = \mathbf{u}[n] - \hat{\mathbf{G}}[n]\hat{\mathbf{d}}[n]$$
$$\hat{\mathbf{e}}[n] = \mathbf{G}[n]\mathbf{d}[n]+\mathbf{v}[n]-\hat{\mathbf{G}}[n]\mathbf{d}[n] = \Gamma[n]\mathbf{d}[n]+\mathbf{v}[n]$$

- Assume noise statistics slowly varying and calculate correlation of estimate noise vec.

$$\gamma_i[n] = \frac{1}{L-1-i}\sum_{j=0}^{L-1-i} e[n+i+j]e^*[n+j], \quad i = 0, \ldots, L-1$$

- RLS Update: $\hat{\mathbf{R}}_{0,[1,i]}[n] = \sum_{k=0}^{n} \lambda\gamma_i[k]$

Additional Question:

How does channel length estimation effect equalization performance?

Recall DFE Equations

- DFE Eq: $\hat{d}[n] = \hat{\mathbf{h}}_{\text{opt}}^H[n]\mathbf{z}[n] = \mathbf{h}_{\text{ff}}^H\mathbf{u}[\mathbf{n}] + \mathbf{h}_{\text{fb}}^H\hat{\mathbf{d}}_{\text{fb}}$

Solution to Weiner-Hopf Eq.

$$\begin{aligned} \hat{\mathbf{h}}_{\text{opt}}[n] &= \mathbf{R_z}^{-1}[n]\mathbf{r}_{\mathbf{zd}}[n] \\ \mathbf{R_z}[n] &= \text{E}\{\mathbf{z}\mathbf{z}^H\} \\ \mathbf{r}_{\mathbf{zd}} &= \text{E}\{\mathbf{z}d^*\} \end{aligned}$$

MMSE Sol. Using Channel Model

$$\begin{aligned} \mathbf{h}_{\text{ff}} &= [\mathbf{G}_0\mathbf{G}_0 + \mathbf{R_v}]^{-1}\mathbf{g}_0 \\ \mathbf{h}_{\text{fb}} &= -\mathbf{G}_{\text{fb}}\mathbf{h}_{\text{ff}} \end{aligned}$$

- Vector of data RX data and TX data est.

$$\mathbf{z}[n] = [u[n-Lc+1] \ \dots \ u[n] \ \dots \ u[n+L_a], \ \hat{d}[n-1] \ \dots \ \hat{d}[n-L_{fb}]]^T$$

- Cost Function: $\hat{\mathbf{h}}_{\text{opt}} = \underset{\mathbf{h}'}{\arg\min}\,\text{E}\{|\mathbf{h}^H\mathbf{z} - d|^2\}$

Channel Length Mismatch

- Model: True channel is estimate + offset

$$\mathbf{G} = \hat{\mathbf{G}} + \Delta\mathbf{G}$$

- *Example*: Static Channel
  - True Channel length = 3
  - Est. Channel Length = 2

$$\hat{\mathbf{G}} = \begin{bmatrix} g[1] & g[0] & 0 \\ 0 & g[1] & g[0] \end{bmatrix}$$

$$\Delta\mathbf{G} = \begin{bmatrix} g[2] & 0 & 0 & 0 \\ 0 & g[2] & 0 & 0 \end{bmatrix}$$

$$\mathbf{G} = \begin{bmatrix} \mathbf{0} & \hat{\mathbf{G}} \end{bmatrix} + \Delta\mathbf{G}$$

DFE: Channel Estimation Errors

Split opt. DFE into estimate plus offset:

$$\begin{aligned} \mathbf{h}_{\mathrm{ff}} &= \hat{\mathbf{h}}_{\mathrm{ff}} + \delta\mathbf{h}_{\mathrm{ff}} \\ \mathbf{h}_{\mathrm{fb}} &= \hat{\mathbf{h}}_{\mathrm{fb}} + \delta\mathbf{h}_{\mathrm{fb}} \end{aligned}$$

Form of equalizer (from estimated channel):

$$\hat{\mathbf{h}}_{\mathrm{ff}} = [\mathbf{G}_0\mathbf{G}_0^H - \mathbf{G}_0\boldsymbol{\Delta}\mathbf{G}_0^H - \boldsymbol{\Delta}\mathbf{G}_0\mathbf{G}_0^H - \boldsymbol{\Delta}\mathbf{G}_0\boldsymbol{\Delta}\mathbf{G}_0^H + \mathbf{R}_{\mathbf{v}}]^{-1}\hat{\mathbf{g}}_0$$

$$\hat{\mathbf{h}}_{fb} = -(\mathbf{G}_{fb} - \boldsymbol{\Delta}\mathbf{G}_{fb})(\mathbf{h}_{ff} - \delta\mathbf{h}_{ff})$$

Form of equalizer offset:

$$\delta\mathbf{h}_{ff} = \mathbf{Q}'^{-1}[\mathbf{I} - \mathbf{W}'\mathbf{Q}'^{-1}]^{-1}\mathbf{W}'\mathbf{Q}'^{-1}\mathbf{g}_0$$

$$\delta\mathbf{h}_{fb} = \boldsymbol{\Delta}\mathbf{G}_{fb}(\hat{\mathbf{h}}_{ff}) - \mathbf{G}_{fb}\delta\mathbf{h}_{ff}$$

$$\mathbf{Q}' = [\mathbf{G}_0\mathbf{G}_0^H + \mathbf{R}_{\mathbf{v}}]$$

$$\mathbf{W}' = \mathbf{G}_0\boldsymbol{\Delta}\mathbf{G}_0^H + \boldsymbol{\Delta}\mathbf{G}_0\mathbf{G}_0^H + \boldsymbol{\Delta}\mathbf{G}_0\boldsymbol{\Delta}\mathbf{G}_0^H$$

DFE: Mean squared error analysis

Estimated DFE error: $\hat{e}_{\text{dfe}} = \hat{\mathbf{h}}_{ff}\mathbf{u} + \hat{\mathbf{h}}_{fb}\hat{\mathbf{d}}_{fb}$

Estimated expected squared error: $\mathrm{E}\{|\hat{e}_{\text{dfe}}|^2\} = \mathrm{E}\{|\hat{\mathbf{h}}_{ff}\mathbf{u} + \hat{\mathbf{h}}_{fb}\hat{\mathbf{d}}_{fb} - d|^2\}$

Estimated expected squared error (Channel Form):

$$\mathrm{E}\{|\hat{e}_{\text{dfe}}|^2\} = \sigma^2_{0,dfe} + \hat{\mathbf{h}}_{ff}^H(\mathbf{\Delta G}_{fb}\mathbf{\Delta G}_{fb}^H)\hat{\mathbf{h}}_{ff} + \delta\mathbf{h}_{ff}^H[\mathbf{G}_0\mathbf{G}_0^H + \mathbf{R}_{\mathbf{v}}]^{-1}\delta\mathbf{h}_{ff}$$

Estimated expected squared error (Excess Error Form):

$$\mathrm{E}\{|\hat{e}_{\text{dfe}}|^2\} =$$

$$\sigma^2_{0,dfe} + \hat{\mathbf{h}}_{ff}^H(\mathbf{\Delta G}_{fb}\mathbf{\Delta G}_{fb}^H)\hat{\mathbf{h}}_{ff} + \mathbf{h}_{\mathrm{ff}}{}^H\mathbf{W}'^H[\mathbf{I} - \mathbf{W}'\mathbf{Q}'^{-1}]^{-H}\mathbf{Q}'^{-1}[\mathbf{I} - \mathbf{W}'\mathbf{Q}'^{-1}]^{-1}\mathbf{W}'\mathbf{h}_{\mathrm{ff}}$$

MAE

Excess Error

DFE: Offset Est. and Compensation

1. Estimate error vector (same as for LE)

$$\epsilon_{\text{dfe}} = \hat{\mathbf{G}}\hat{\mathbf{d}} - \mathbf{u}$$

2. Outer product w/ extended data vector

$$\mathrm{E}\{\epsilon_{\text{dfe}}\mathbf{d}'^{H}\} = \boldsymbol{\Delta}\mathbf{G}'$$

3. Subtract estimated channel offset

$$\epsilon_{\text{dfe}}{}' = \epsilon_{\text{dfe}} - \boldsymbol{\Delta}\mathbf{G}'\mathbf{d}'$$

4. Split Estimated Channel offset into FB and other

$$\boldsymbol{\Delta}\mathbf{G}' \begin{cases} \nearrow \boldsymbol{\Delta}\mathbf{G}_0' \\ \searrow \boldsymbol{\Delta}\mathbf{G}_{fb}' \end{cases}$$

5. Plug values into equalizer equation

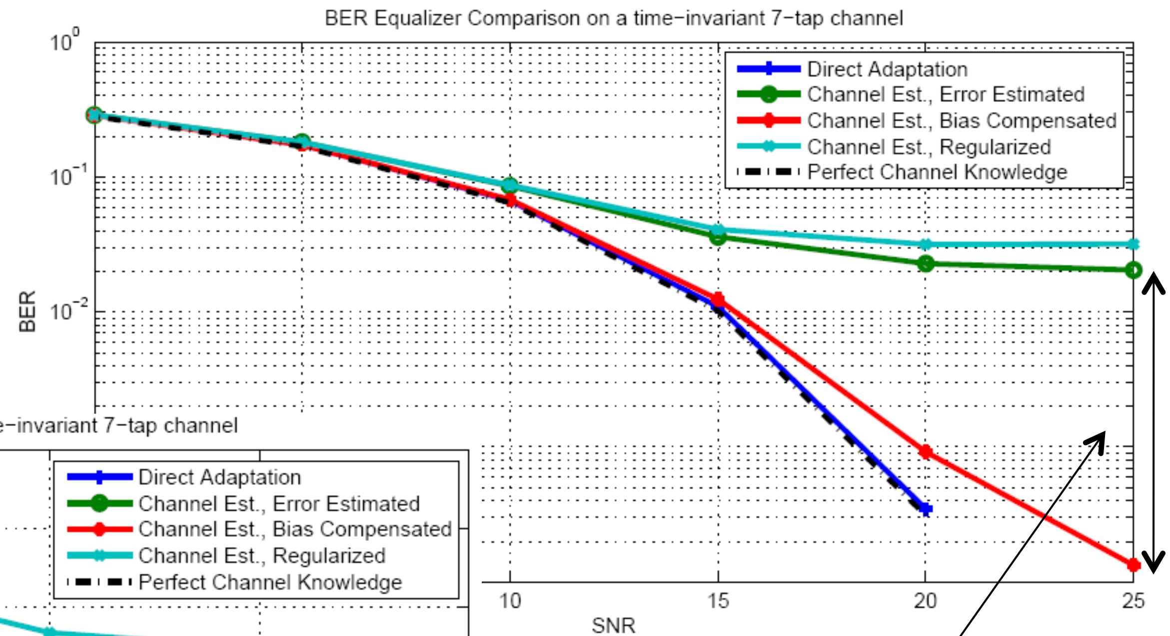
$$\hat{\mathbf{h}}'_{ff} = [\epsilon_{\text{dfe}}{}'\epsilon_{\text{dfe}}{}'^{H} + \hat{\mathbf{G}}_0\boldsymbol{\Delta}\mathbf{G}_0'^{H} + \boldsymbol{\Delta}\mathbf{G}_0'\hat{\mathbf{G}}_0^{H} + \boldsymbol{\Delta}\mathbf{G}_0'\boldsymbol{\Delta}\mathbf{G}_0'^{H} + \hat{\mathbf{G}}_0\hat{\mathbf{G}}_0]^{-1}\mathbf{g}_0$$

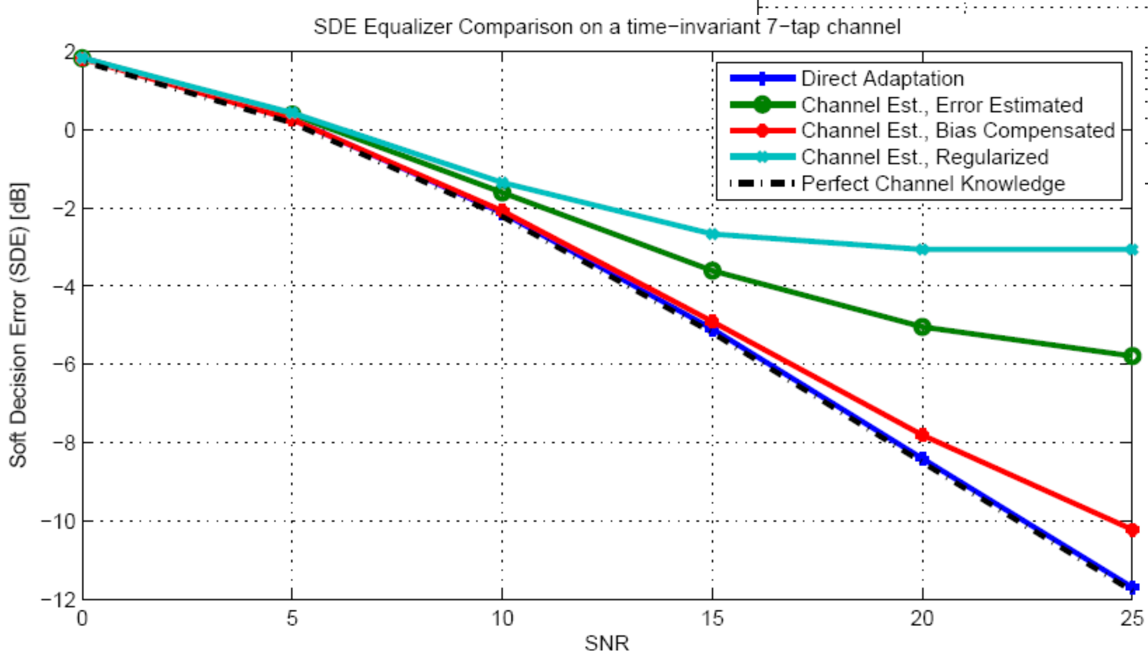
$$\hat{\mathbf{h}}'_{\text{fb}} = -[\hat{\mathbf{G}}_{fb} + \boldsymbol{\Delta}\mathbf{G}'_{fb}]^{H}\hat{\mathbf{h}}'_{ff}$$

Simulation: DFE Time-Invariant Channel

Simulation Parameters:

- True Channel Length = 7
- Est. Channel Length = 6
- Equalizer = DFE
  - L_{fb} = 5

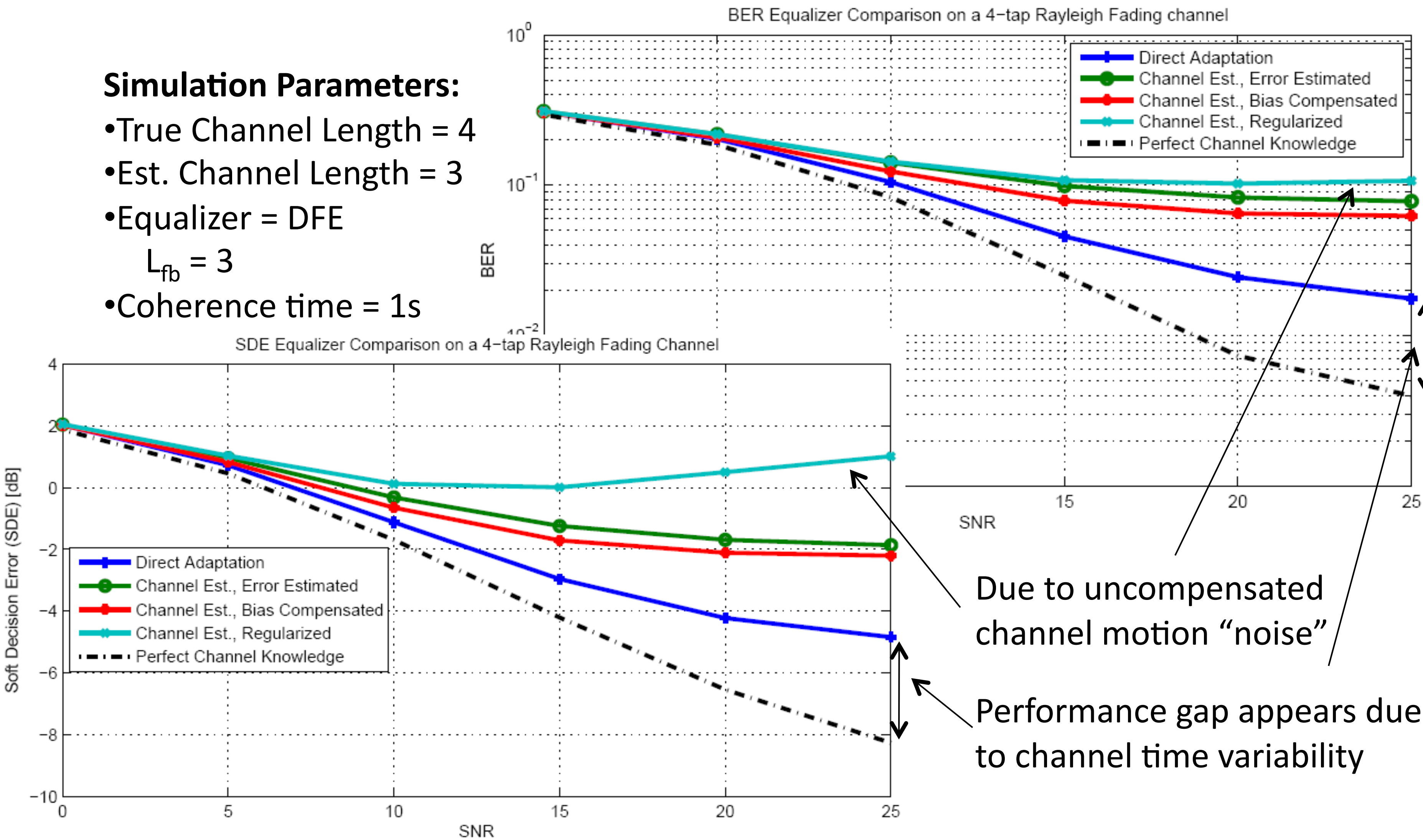




Performance gap due to feedback path

Simulation: DFE Rayleigh Fading Channel

Simulation Parameters:
- True Channel Length = 4
- Est. Channel Length = 3
- Equalizer = DFE
  L_{fb} = 3
- Coherence time = 1s

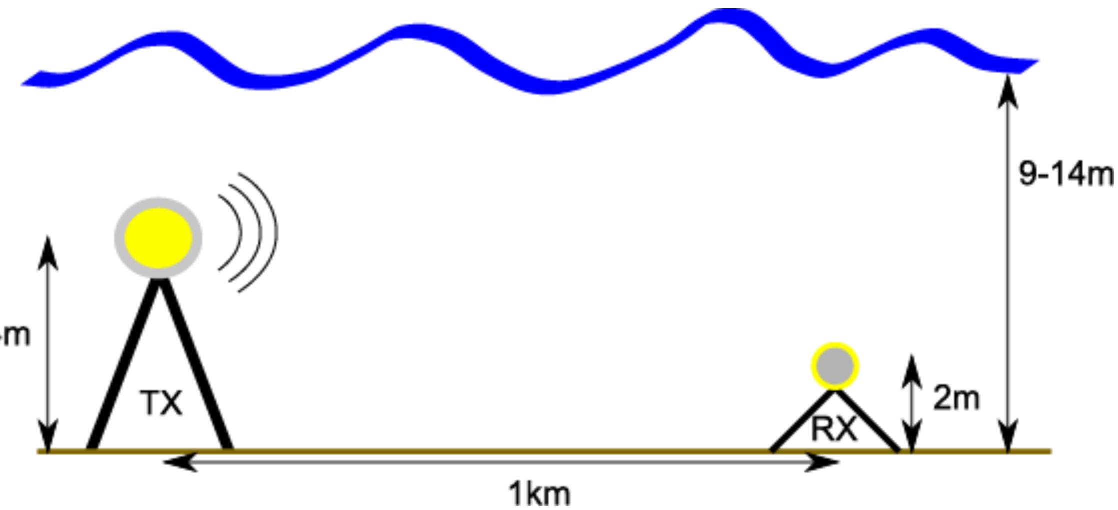


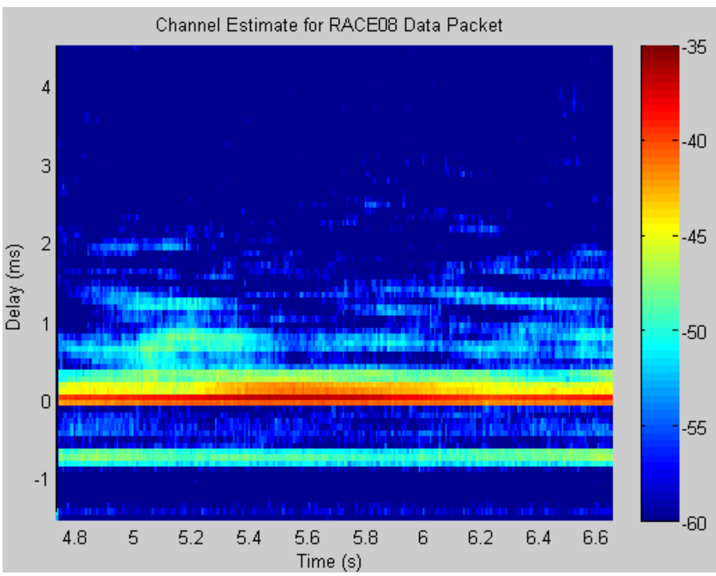
Due to uncompensated channel motion “noise”

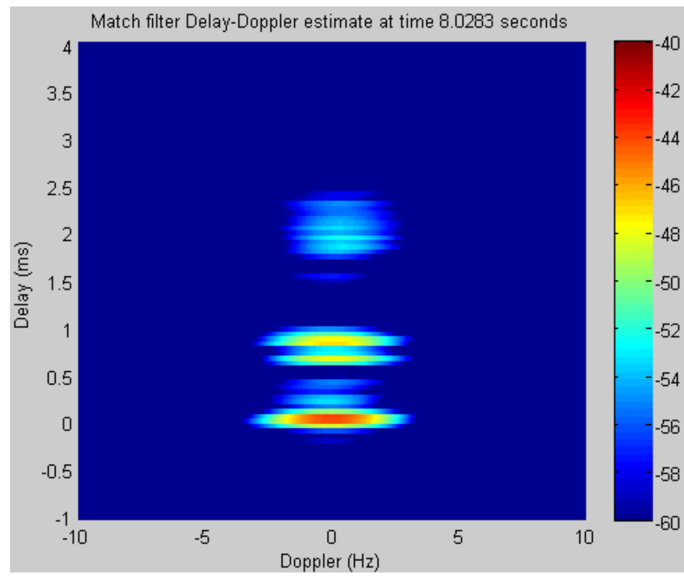
Performance gap appears due to channel time variability

Experimental Setup: RACE08

Experiment Signal Parameters:
- 12 kHz carrier
- 6510 ksym/s (~6 kHz bandwidth)
- BPSK encoding
- Used 1 receiving element (of 12)
- 39062.5 samples / second







Testing Setup: RACE08

- Channel Est. Parameters:

  $N_a = 2, N_c = 6$

- Equalizer = DFE

  $L_a = 5, L_c = 3$

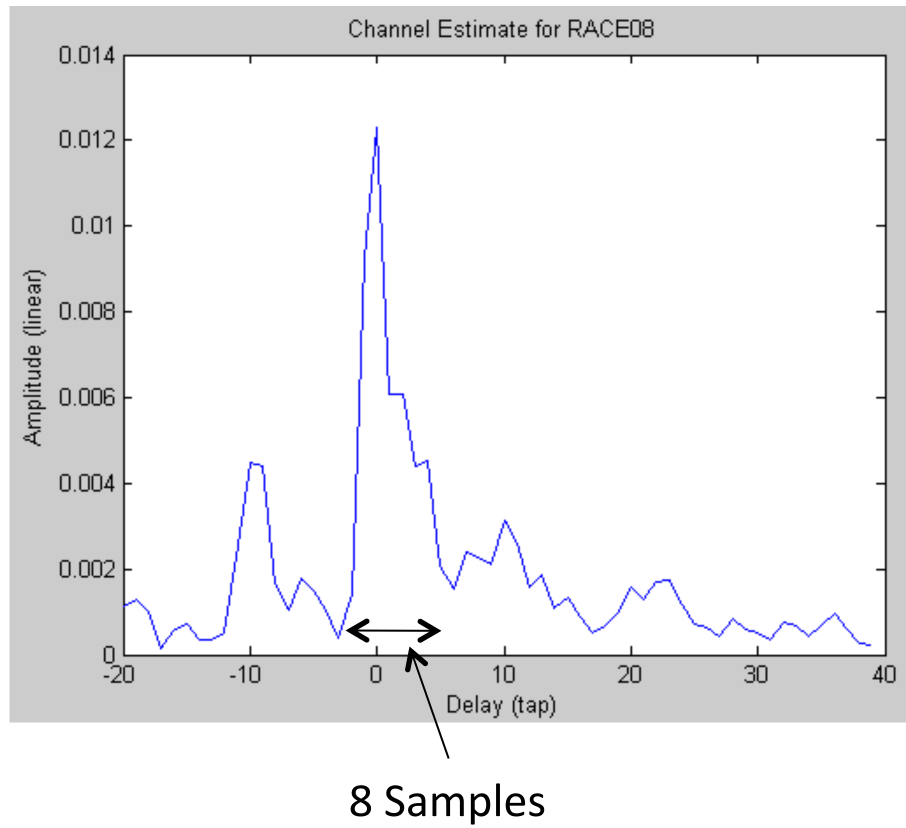
- Packet Length:

  25000 sym

- RLS Parameters:

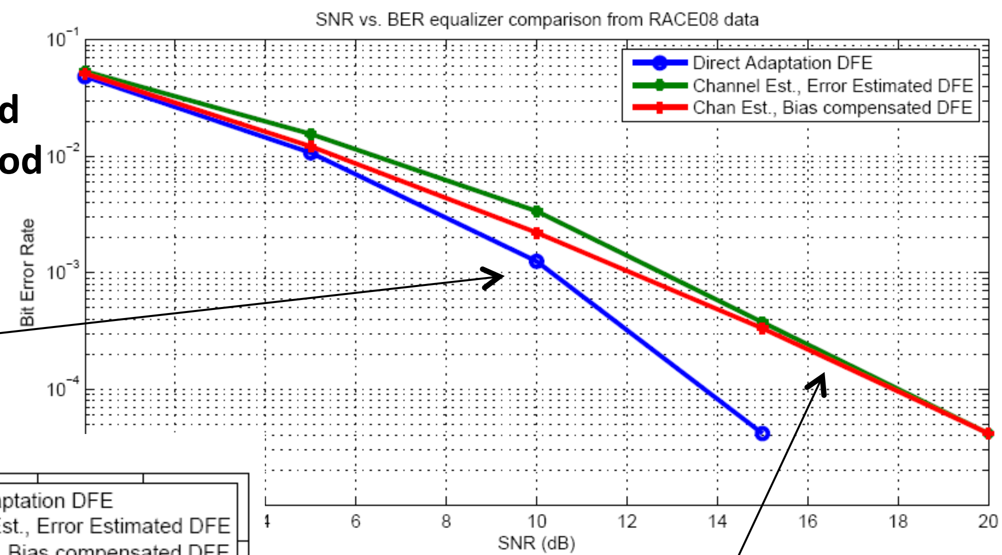
  $\lambda=0.996$

  $N_{train} = 1000$ sym

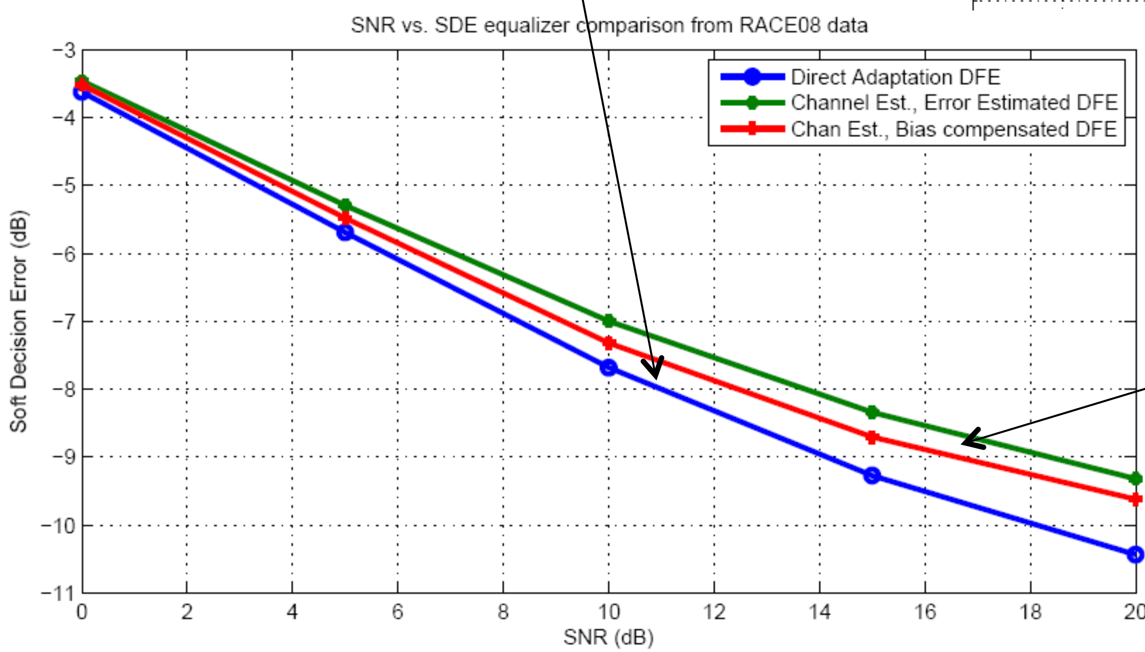


Experimental Results: RACE08

Direct Adaptation DFE = standard DA DFE
***Chan. Est., Error Estimated DFE* = Previous method**
***Chan Est., Biased Removed DFE* = Proposed Method**



Direct Adaptation outperforms others



Proposed Method (Biased Removed) outperforms error est. DFE

Take-home Message

- Effect of channel length mismatch is proportional to energy in channel that is not modeled

- DA equalization does not suffer from bad channel length information
  - No way to include information in algorithm

- Can recover some of the lost energy adaptively