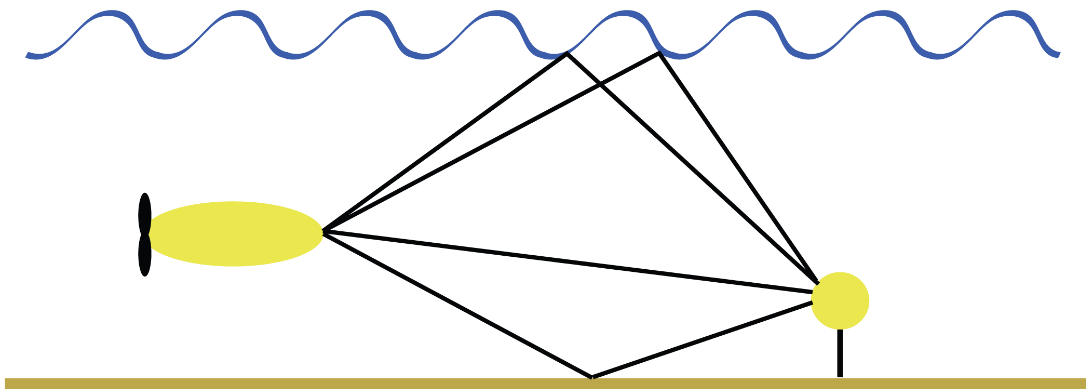
Comparison and Analysis of Equalization Techniques for the Time-Varying Underwater Acoustic Channel



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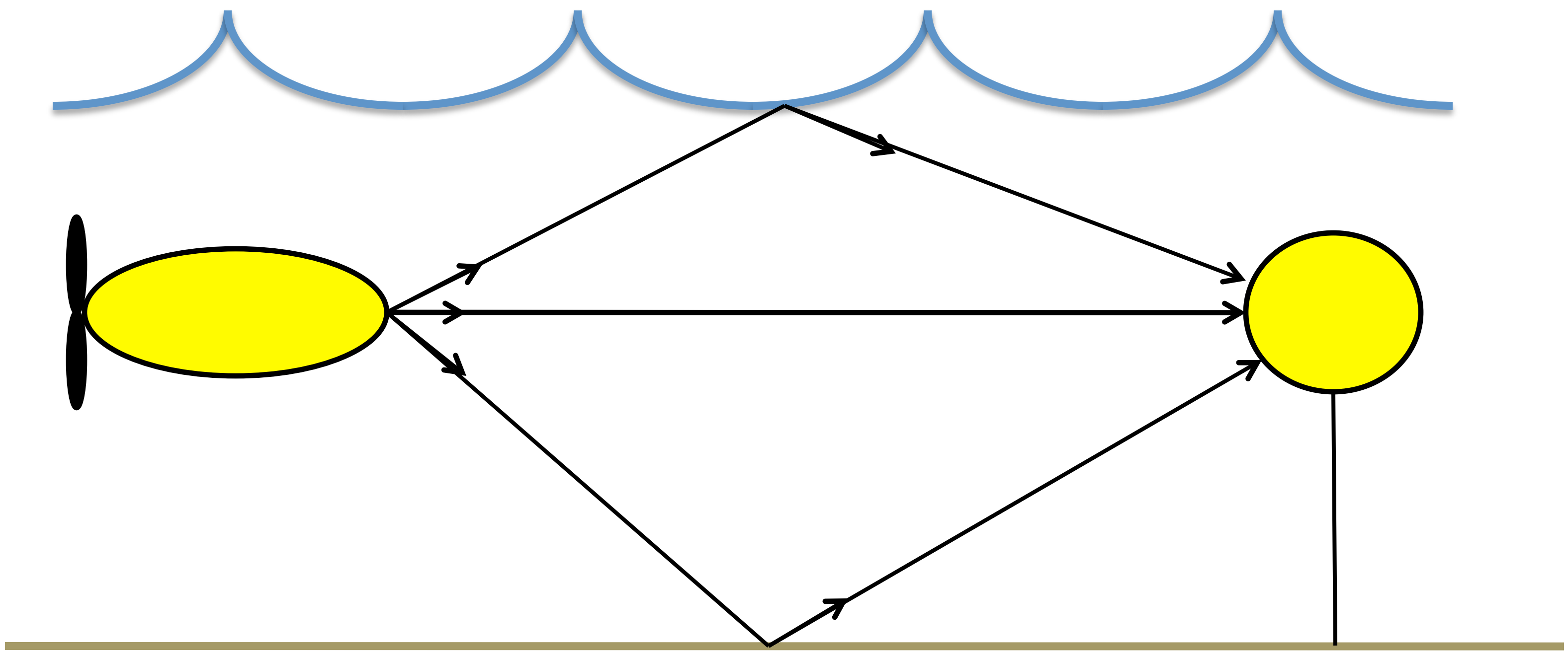
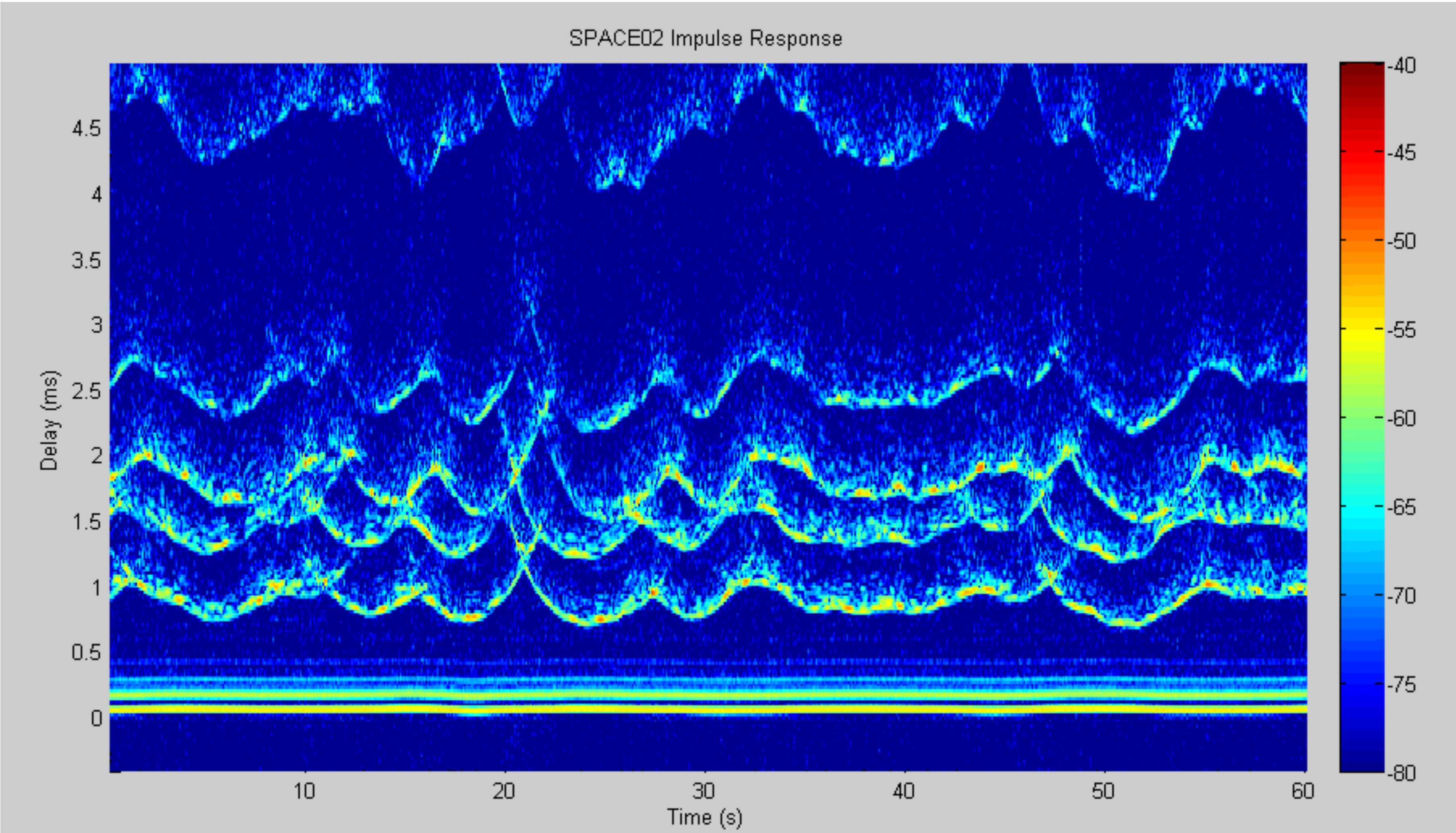
Outline

- Introduction:
  - – Underwater Communication
  - – Decision Feedback Equalization
    - Channel Estimate Based
    - Direct Adaptation
- Analysis of Equalization Behavior
- Simulation Results
- Summary and Conclusion
- Future Directions

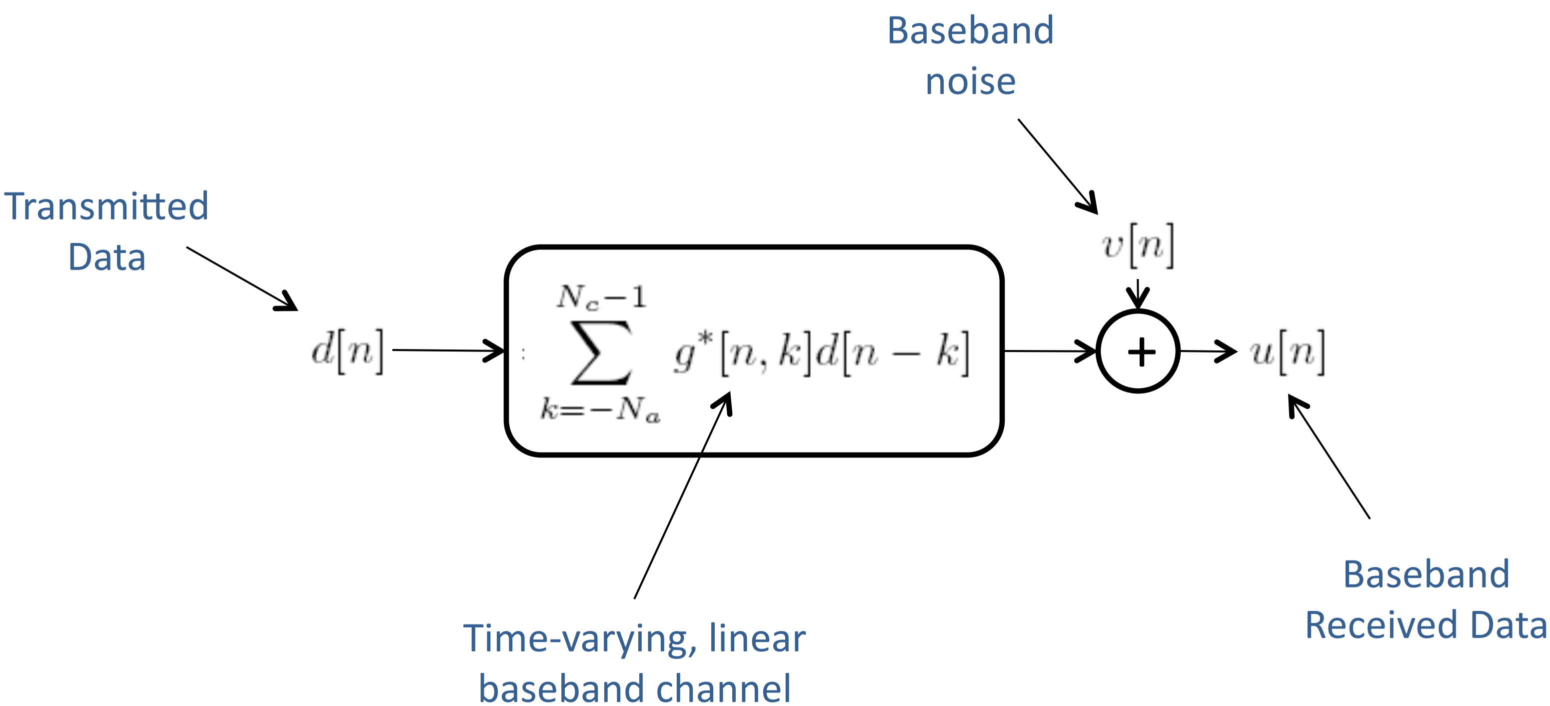
Underwater Communication



Time Varying Impulse Response



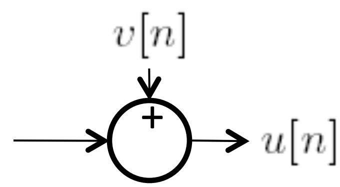
Channel Model



Channel Model (cont.)

Vector-form: $\mathbf{g}^H[n]\mathbf{d}[n] + v[n] = u[n]$

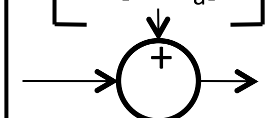
$$\begin{bmatrix} g^*[n,-N_c+1] \; g^*[n,N_c+2] \; \ldots \; g^*[n,0] \; \ldots \; g^*[n,N_a] \end{bmatrix} \begin{bmatrix} d[n-N_c+1] \\ \ldots \\ d[n] \\ \ldots \\ d[n+N_a] \end{bmatrix}$$



Matrix Vector-form: $\mathbf{G}[n]\mathbf{d}[n] + \mathbf{v}[n] = \mathbf{u}[n]$

$$\begin{bmatrix} g^*[n-L_c+1,N_c-1] \ldots g^*[n-L_c+1,0] \ldots g^*[n-L_c+1,N_a] \; 0 \; 0 \ldots 0 \\ 0 \; g^*[n-L_c+2,N_c-1] \ldots g^*[n-L_c+2,0] \ldots g^*[n-L_c+2,N_a] \; 0 \ldots 0 \\ \ldots \\ 0 \ldots 0 \; g^*[n+L_a-1,N_c-1] \ldots g^*[n-L_a-1,0] \ldots g^*[n+L_a-1,N_a] \; 0 \\ 0 \ldots 0 \; 0 \; 0 \; 0 \; 0 \; g^*[n+L_a,N_c-1] \ldots g^*[n-L_a,0] \ldots g^*[n+L_a,N_a] \end{bmatrix} \begin{bmatrix} d[n-L_c-N_c+1] \\ d[n-L_c-N_c+2] \\ \ldots \\ d[n] \\ \ldots \\ d[n+L_a+N_a-1] \\ d[n+L_a+N_a] \end{bmatrix}$$

$$\begin{bmatrix} v[n-L_c+1] \\ \ldots \\ v[n] \\ \ldots \\ v[n+L_a] \end{bmatrix} \qquad \begin{bmatrix} u[n-L_c+1] \\ \ldots \\ u[n] \\ \ldots \\ u[n+L_a] \end{bmatrix}$$



Equalization

TX Data bit (linear) estimator: $\hat{d}[n] = \mathbf{h}^H[n]\mathbf{z}[n]$ ← Vector of RX data and TX data estimates

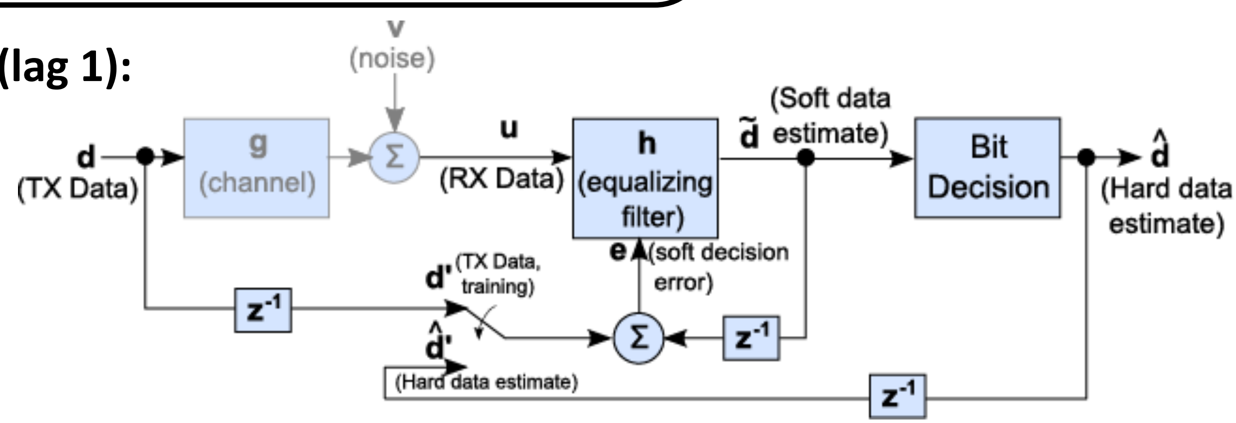
LMMSE Optimization: $\hat{\mathbf{h}}_{\text{opt}} = \arg\min_{\mathbf{h}'} \mathrm{E}\{|\mathbf{h}^H\mathbf{z} - d|^2\}$

Solution: $\hat{\mathbf{h}}_{\text{opt}}[n] = \mathbf{R_z}^{-1}[n]\mathbf{r_{zd}}[n]$

$$\mathbf{R_z}[n] = \mathrm{E}\{\mathbf{z}\mathbf{z}^H\}$$

$$\mathbf{r_{zd}} = \mathrm{E}\{\mathbf{z}d^*\}$$

Recursive Processing (lag 1):



Decision Feedback Equalizer (DFE)

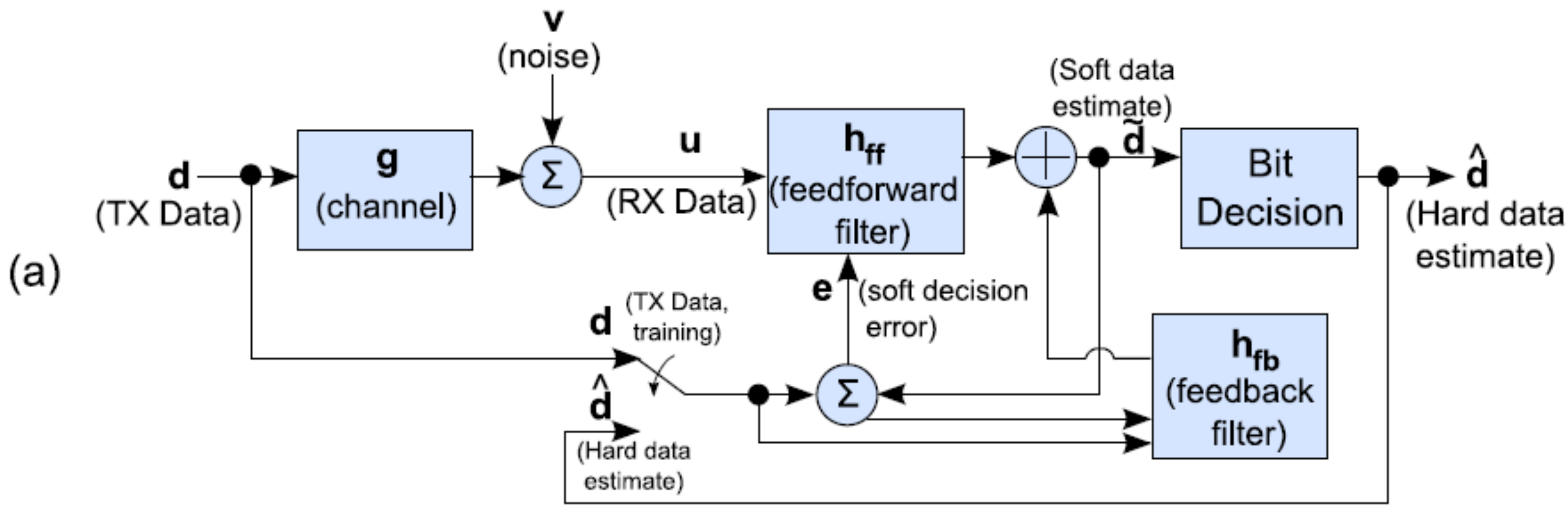
- Two Parts:
  - (Linear) feed-forward filter (of RX data)
  - (Linear) feedback filter (of data estimates)
- Estimate using RX data and TX data estimates

$$\mathbf{z}[n] \;=\; [u[n-Lc+1] \;\ldots\; u[n] \;\ldots\; u[n+L_a],\; \hat{d}[n-1] \;\ldots\; \hat{d}[n-L_{fb}]]^T$$

- Split Channel convolution Matrix: $\mathbf{G} \to \mathbf{G}_0,\ \mathbf{G}_{fb}$
  - Received data becomes: $\mathbf{u} = \mathbf{G}_0\mathbf{d}_0 + \mathbf{G}_{fb}\mathbf{d}_{fb} + \mathbf{v}$
- Minimum Achievable Error:

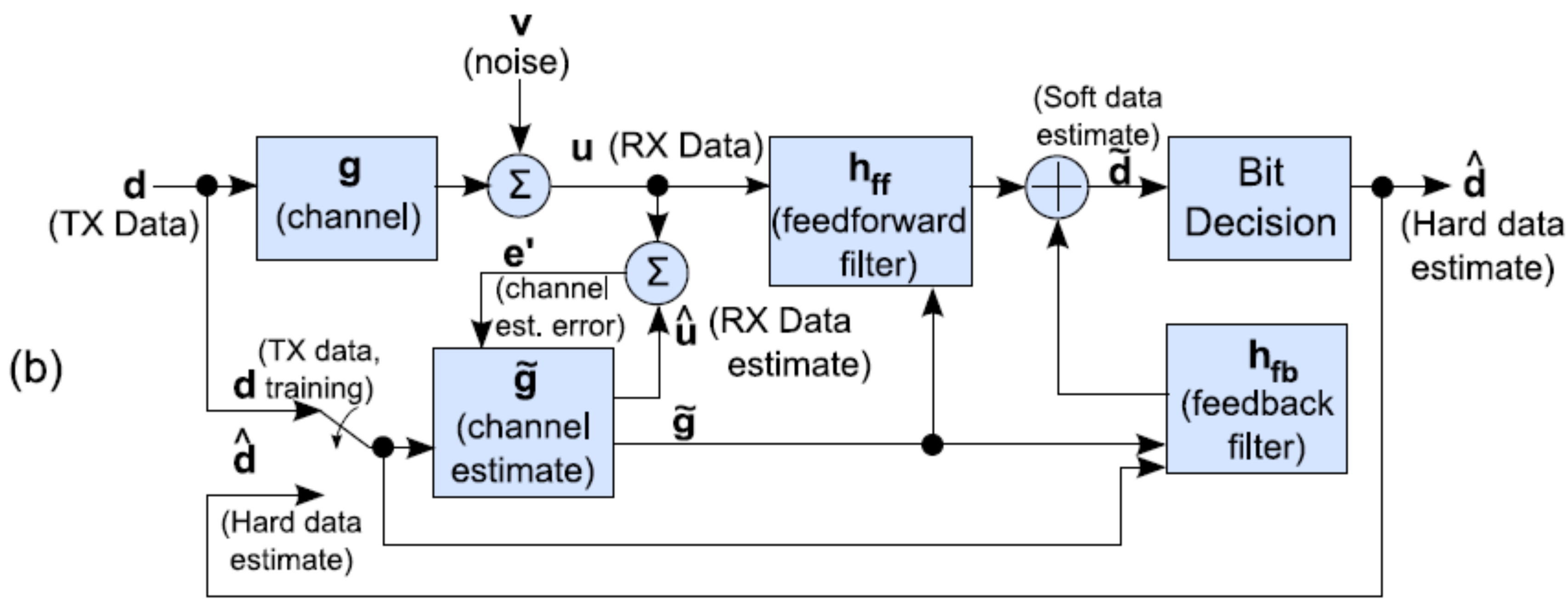
$$\sigma^2_{0,dfe} = 1 - \mathbf{g}_0^H[\mathbf{G}_0\mathbf{G}_0^H + \mathbf{R}_{\mathbf{v}}]^{-1}\mathbf{g}_0$$

DFE: Direct Adaptation



$$\mathrm{h} = \begin{bmatrix} \mathbf{h}_{ff}[n] \\ \mathbf{h}_{fb}[n] \end{bmatrix} = \mathrm{E}\{\mathbf{z}\mathbf{z}^H\}^{-1}\mathrm{E}\{\mathbf{z}d^*\}$$

DFE: Channel Estimate



$$\mathbf{h_{ff}} = [\mathbf{G}_0\mathbf{G}_0^H + \mathbf{R_v}]^{-1}\mathbf{Gs}$$

$$\mathbf{h_{fb}} = -\mathbf{G}_{fb}^H\mathbf{h_{ff}}$$

Assumptions

- Unit variance, white transmit data

$$E\{\mathbf{d}[n]\mathbf{d}^H[n]\} = \mathbf{I}$$

- TX data and obs. noise are uncorrelated

$$E\{\mathbf{v}[n]\mathbf{d}^H[m]\} = \mathbf{0}$$

  – Obs. Noise variance:

$$\mathbf{R_v} = E\{\mathbf{v}[n]\mathbf{v}^H[n]\}$$

- Perfect data estimation (for feedback)

$$\hat{\mathbf{d}} = \mathbf{d}$$

- Equalizer Length = Estimated Channel Length

$$N_a + N_c = L_a + L_c$$

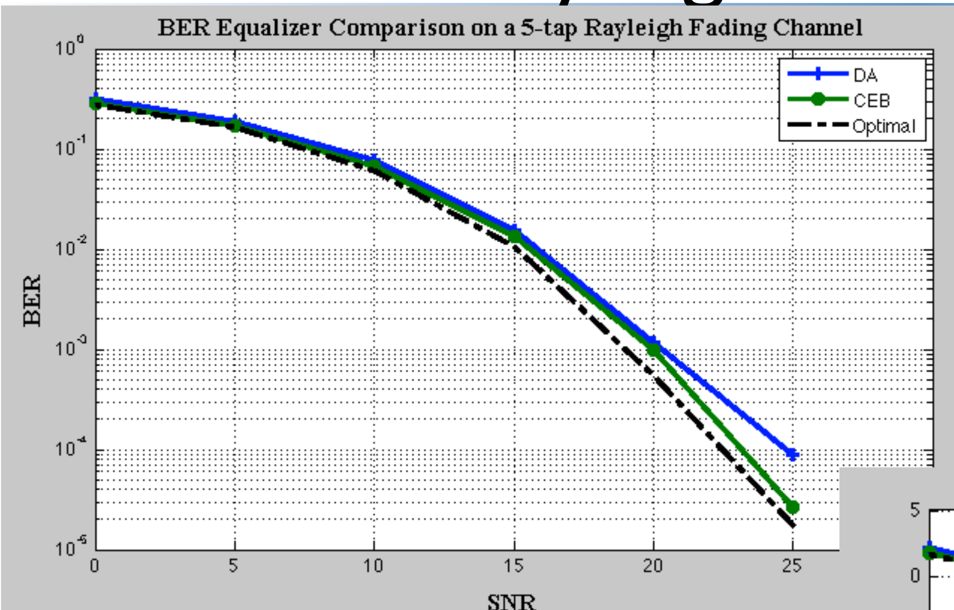
- MMSE Equalizer Coefficients have form:

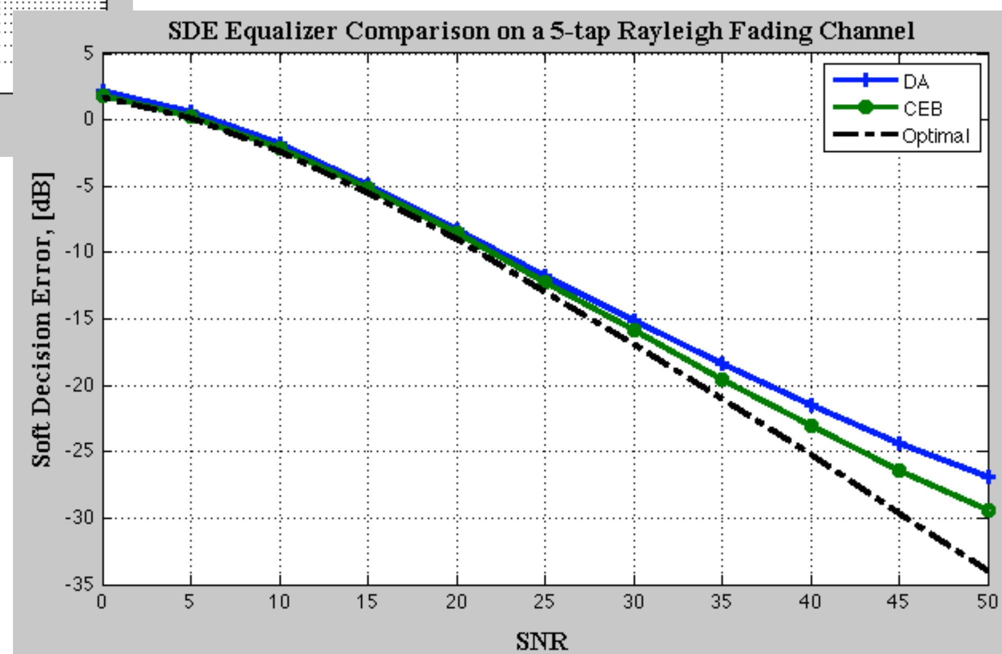
$$\mathbf{h_{ff}} = [\mathbf{G}_0\mathbf{G}_0^H + \mathbf{R_v}]^{-1}\mathbf{Gs}$$
$$\mathbf{h_{fb}} = -\mathbf{G}_{fb}^H\mathbf{h_{ff}}$$

Comparison between DA and CEB

- In the past, CEB methods empirically shown to have lower mean squared error at high SNR
- Reasons for difference varied:
  – Condition number of correlation matrix
  – Number of samples required to get good est.
- Analysis to follow: low and high SNR regimes

Comparison of DA and CEB on Rayleigh Fading Channel





Why the difference?

- Correlation time
  - DA equalizer taps have lower correlation time at high SNR
  - At low SNR, two methods equivalent
- But how do we show this?
  - Combination of analysis and simulation

AR channel model

- Simple channel model to analyze
- Similar to encountered situations

$$g[n+1] = \alpha g[n] + w[n]$$

$$R_{gg}[k] = \mathrm{E}\{g[n]g^*[n+k]\} \; = \begin{cases} \sigma_w^2 \left(\frac{(\alpha^*)^k}{1-|\alpha|^2} \right) & k \geq 0 \\ \sigma_w^2 \left(\frac{\alpha^{-k}}{1-|\alpha|^2} \right) & k < 0 \end{cases}$$

Low SNR

$$\begin{aligned}\mathbf{h}_{\mathbf{ff}}[n+1] &= (\mathbf{G}_0[n+1]\mathbf{G}_0^H[n+1] + \mathbf{R}_\mathbf{v})^{-1}(\mathbf{g}_0[n+1]) \\ &\approx \mathbf{R}_\mathbf{v}{}^{-1}(\alpha\mathbf{g}_0[n] + \mathbf{w}[n]) \\ &\approx \alpha\mathbf{h}_{\mathbf{ff}}[n] + \mathbf{R}_\mathbf{v}{}^{-1}\mathbf{w}[n]\end{aligned}$$

$$\mathbf{R}_\mathbf{v} + \mathbf{G}[n]\mathbf{G}^H[n] \approx \mathbf{R}_\mathbf{v}$$

High SNR

$$\mathbf{h}_{\mathbf{ff}}[n+1] \;\; = \;\; (\mathbf{G}_0[n+1]\mathbf{G}_0^H[n+1] + \mathbf{R}_{\mathbf{v}})^{-1}(\mathbf{g}_0[n+1])$$

$$\mathbf{Q}[n] = \mathbf{G}_0[n]\mathbf{G}_0^H[n] + \mathbf{R}_{\mathbf{v}} \approx \mathbf{G}_0[n]\mathbf{G}_0^H[n]$$

$$(\mathbf{G}_0[n]\mathbf{G}_0^H[n])\mathbf{h}_{\mathbf{ff}}[n] = \mathbf{g}_0[n]$$

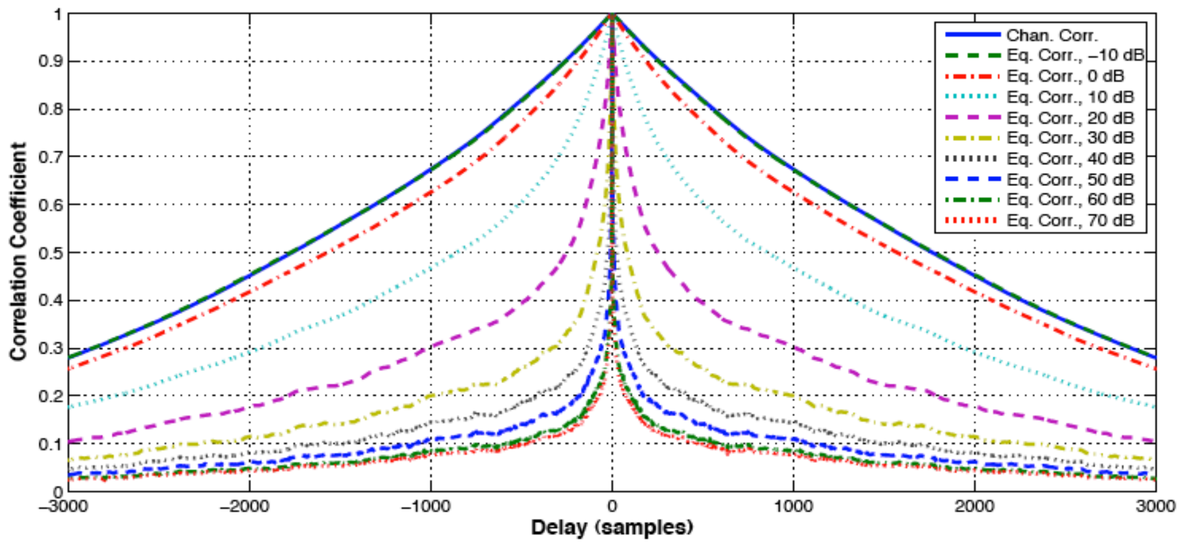
$$(\mathbf{G}_0[n]\mathbf{G}_0^H[n])\mathbf{h}_{\mathbf{ff}}[n] = \mathbf{g}_0[n]$$

$$G_0 = \begin{bmatrix} g_0^*[n-L+1] & 0 & 0 & \cdots & 0 \\ g_1^*[n-L+2] & g_0^*[n-L+2] & 0 & \cdots & 0 \\ g_2^*[n-L+3] & g_1^*[n-L+3] & g_0^*[n-L+3] & \cdots & 0 \\ \vdots & & & \ddots & \vdots \\ g_L^*[n] & g_{L-1}^*[n] & g_{L-2}^*[n] & \cdots & \mathbf{g}_0^*[n] \end{bmatrix}$$

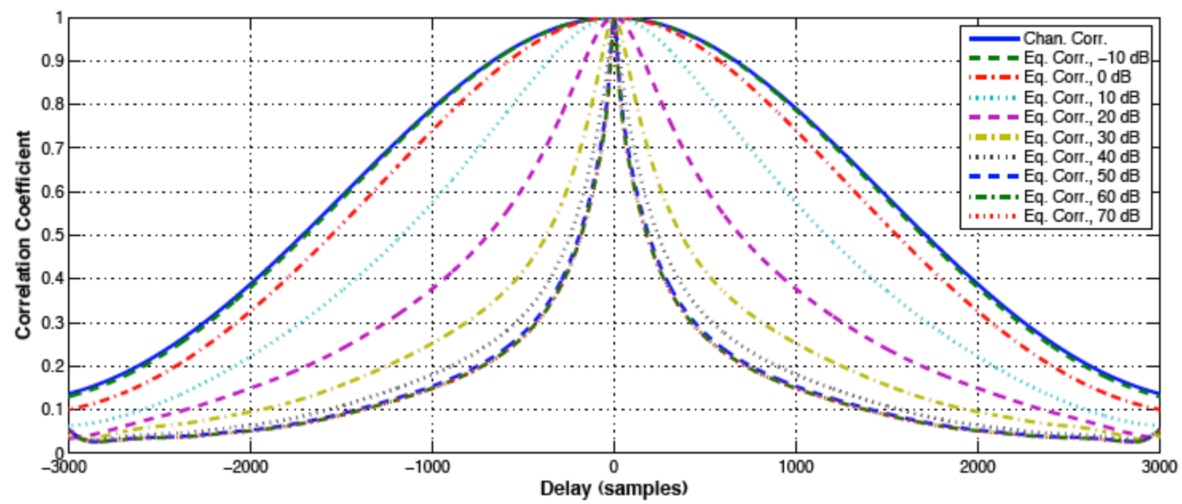
$$\mathbf{G}_0\mathbf{G}_0^H = \begin{bmatrix} |g_0|^2 & \cdots \\ g_0 g_1^* & \cdots \\ g_0 g_2^* & \cdots \\ \vdots & \vdots \\ g_0 g_L & \cdots \end{bmatrix} \qquad \mathbf{h}[n] = \begin{bmatrix} 1/g_0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

Correlation over SNR

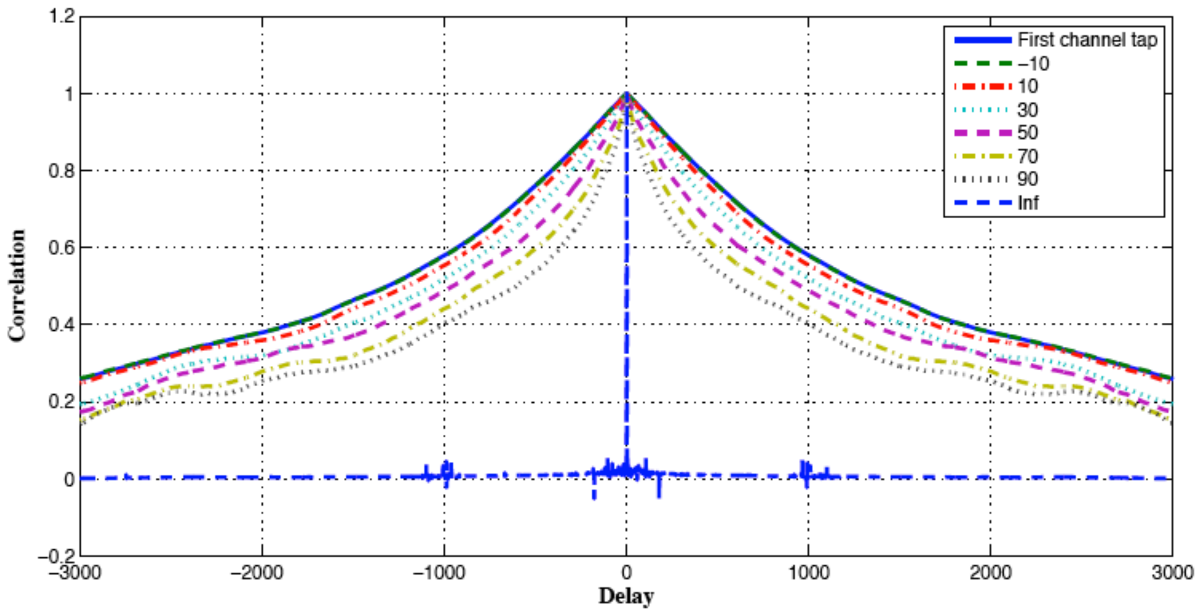
AR(1) model

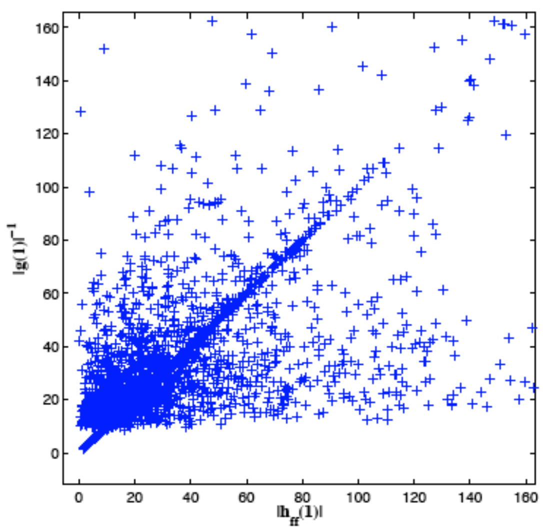


Gaussian model



Multi-tap AR(1) model





Conclusions

- As SNR increases, correlation time of equalizer taps is reduced
  - CEB is tracking value correlated over longer time
  - DA should do worse
- Assumed noise statistics were stationary
  - Not always case in underwater
- Underwater communication is power limited
  - Operate in low SNR regime (<35dB)

Future Work

- Include channel state information into DA
  – Sparsity

- Reduce number of snapshots for channel model
  – Physical constraints?
  – Compressed sensing?

Questions?